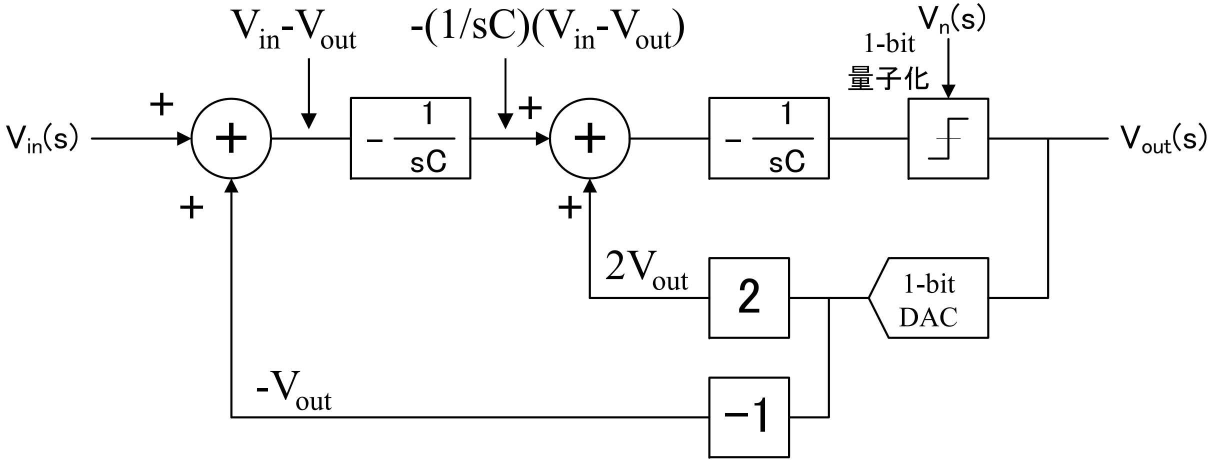
Problems 04

DELTA-SIGMA MODULATORS

Problem 1-1



$$V_{out} = -\frac{1}{sC}\left\{-\frac{1}{sC}(V_{in} - V_{out}) + 2V_{out}\right\} + V_n$$

$$(1 + \frac{2}{sC} + \frac{1}{(sC)^2})V_{out} = \frac{1}{(sC)^2}V_{in} + V_n$$

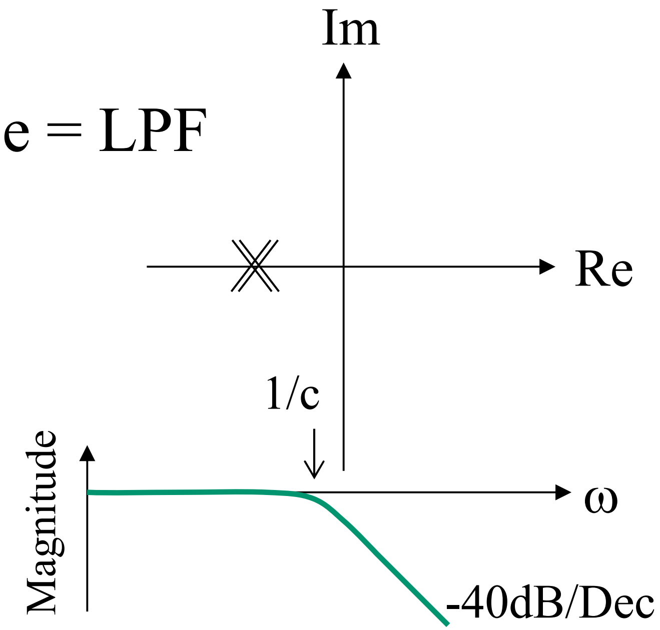
$$V_{out} = \frac{1}{(sC+1)^2}V_{in} + \frac{(sC)^2}{(sC+1)^2}V_n$$

Problem 1-2

$$H_s(s) = \frac{1}{(sC+1)^2}$$

Zero	None
Pole	-1/C (Double)

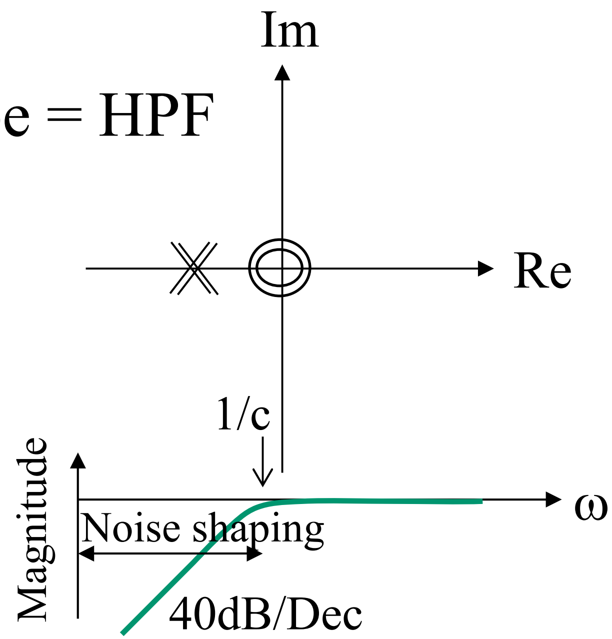
Type = LPF



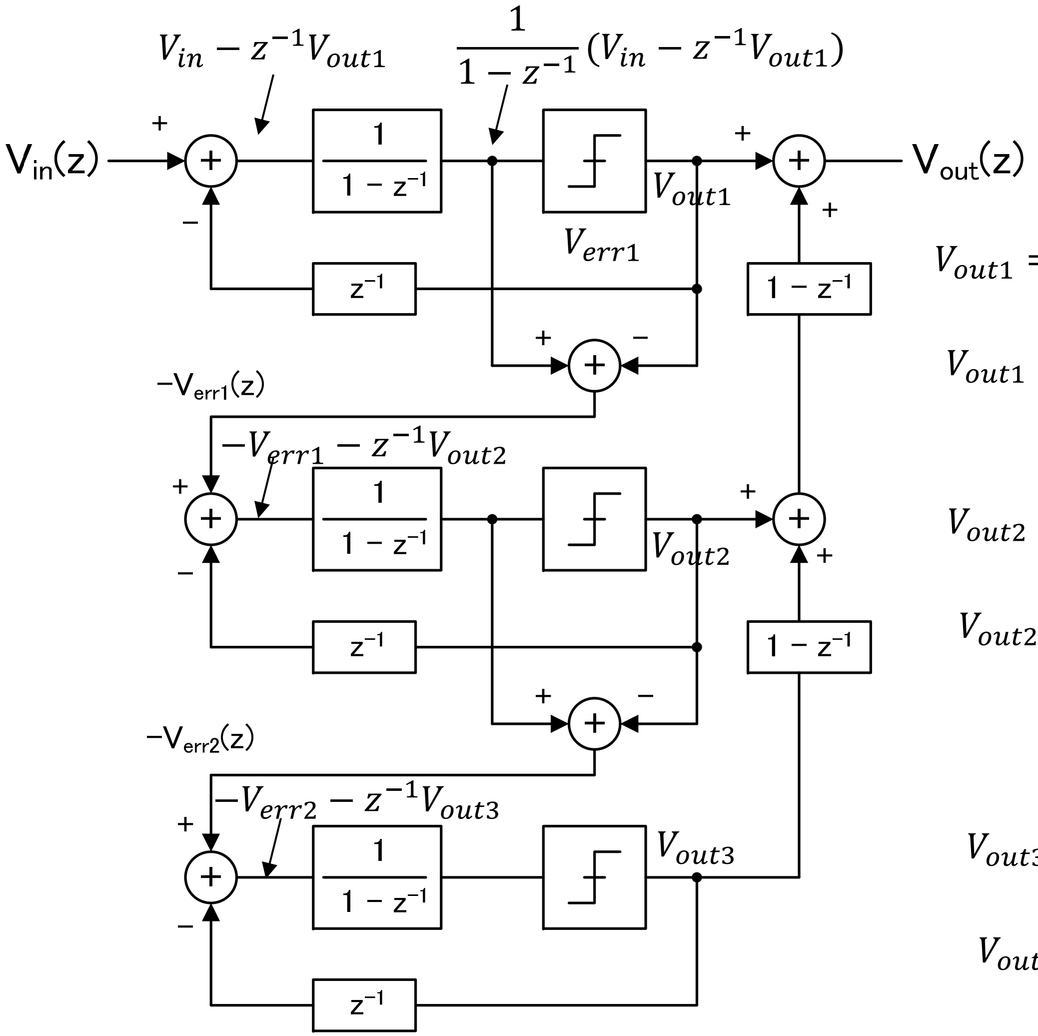
$$H_n(s) = \frac{(sC)^2}{(sC+1)^2}$$

Zero	0 (Double)
Pole	-1/C (Double)

Type = HPF



Problem 2-1



$$V_{out1} = \frac{1}{1-z^{-1}}(V_{in} - z^{-1}V_{out1}) + V_{err1}$$

$$V_{out1} = V_{in} + (1 - z^{-1})V_{err1}$$

$$V_{out2} = \frac{1}{1-z^{-1}}(-V_{err1} - z^{-1}V_{out2}) + V_{err2}$$

$$V_{out2} = -V_{err1} + (1 - z^{-1})V_{err2}$$

$$V_{out3} = \frac{1}{1-z^{-1}}(-V_{err3} - z^{-1}V_{out3}) + V_{err3}$$

$$V_{out3} = -V_{err2} + (1 - z^{-1})V_{err3}$$

Problem 2-2

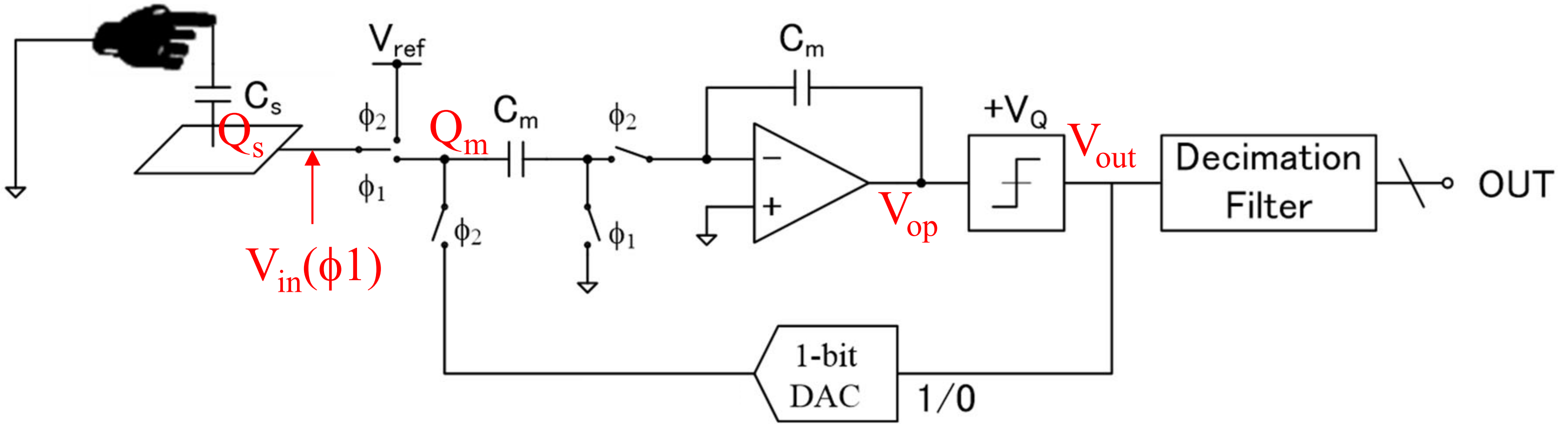
$V_{out} = V_{out1} + (1 - z^{-1})\{V_{out2} + (1 - z^{-1})V_{out3}\}$

$= V_{in} + (1 - z^{-1})V_{err1} + (1 - z^{-1})\{-V_{err1} + (1 - z^{-1})V_{err2} + (1 - z^{-1})\{-V_{err2} + (1 - z^{-1})V_{err3}\}\}$

$= V_{in} + (1 - z^{-1})V_{err1} - (1 - z^{-1})V_{err1} + (1 - z^{-1})^2 V_{err2} - (1 - z^{-1})^2 V_{err2} + (1 - z^{-1})^3 V_{err3}$

$= V_{in} + (1 - z^{-1})^3 V_{err3}$ (3rd order nose shaping)

Problem 3-1



In the phase ϕ_2, $Q_s(t-\frac{T_S}{2}) = C_s V_{ref}$, $Q_m(t-\frac{T_S}{2}) = C_m V_{out}(t-\frac{T_S}{2})$

In the phase ϕ_1, $V_{in}(t) = \frac{1}{C_s + C_m}\left\{Q_s(t-\frac{T_S}{2}) = +Q_m\left(t-\frac{T_S}{2}\right)\right\}$

$= \frac{1}{C_s + C_m}\left\{C_s V_{ref} + C_m V_{out}(t-\frac{T_S}{2})\right\}$ (The temporal changes in the capacitance C_s is slower than the sampling frequency.)

⇩ Z

$$V_{in}(z) = \frac{1}{C_s + C_m}\left\{C_s V_{ref} + z^{-\frac{1}{2}} C_m V_{out}(z)\right\}$$

Problem 3-2

Using the equation shown in Chap.4, Slide 28,

$$V_{op}(z) = \frac{C_m}{C_m}\frac{1}{1-z^{-1}}\left\{z^{-\frac{1}{2}}V_{in}(z) + V_{out}(z)\right\}$$

$$= \frac{1}{1-z^{-1}}\left\{\frac{z^{-\frac{1}{2}}}{C_s + C_m}\left\{C_sV_{ref} + z^{-\frac{1}{2}}C_mV_{out}(z)\right\} - V_{out}(z)\right\}$$

$$= \frac{z^{-\frac{1}{2}}}{1-z^{-1}}\frac{C_s}{C_s + C_m}V_{ref} + \frac{z^{-1}}{1-z^{-1}}\frac{C_m}{C_s + C_m}V_{out}(z) - \frac{1}{1-z^{-1}}V_{out}(z)$$

$$V_{out}(z) = z^{-1}V_{op}(z) + V_Q(z)$$

$$= \frac{z^{-\frac{3}{2}}}{1-z^{-1}}\frac{C_s}{C_s + C_m}V_{ref} + \frac{z^{-2}}{1-z^{-1}}\frac{C_m}{C_s + C_m}V_{out}(z) - \frac{z^{-1}}{1-z^{-1}}V_{out}(z) + V_Q(z)$$

$$\approx \frac{z^{-\frac{3}{2}}}{1-z^{-1}}\frac{C_s}{C_s + C_m}V_{ref} - \frac{z^{-1}}{1-z^{-1}}V_{out}(z) + V_Q(z) \quad (C_s >> C_m)$$

$$V_{out}(z) \approx z^{-\frac{3}{2}}\frac{C_s}{C_s + C_m}V_{ref} + (1 - z^{-1})V_Q(z)$$

問題3.2の補足説明

- この回路は、Capsenseと呼ばれ、タッチセンサや近接センサに使用される
- 参照入力付きDAIを使用しているが、入力が電圧ではなく、C_sに充電された電荷を用いるため、$C_s + C_m$の容量に入力電荷が分配されてしまう問題がある。前ページの計算では、$C_s >> C_m$ の近似を使用したが、この条件下では、$C_s/(C_s + C_m) \fallingdotseq 1$となり、Csの変化を検出する感度が小さくなる
- この問題を解決するために、入力用と帰還用のキャパシタを別々に用意する（第4章スライド28の中段の回路）