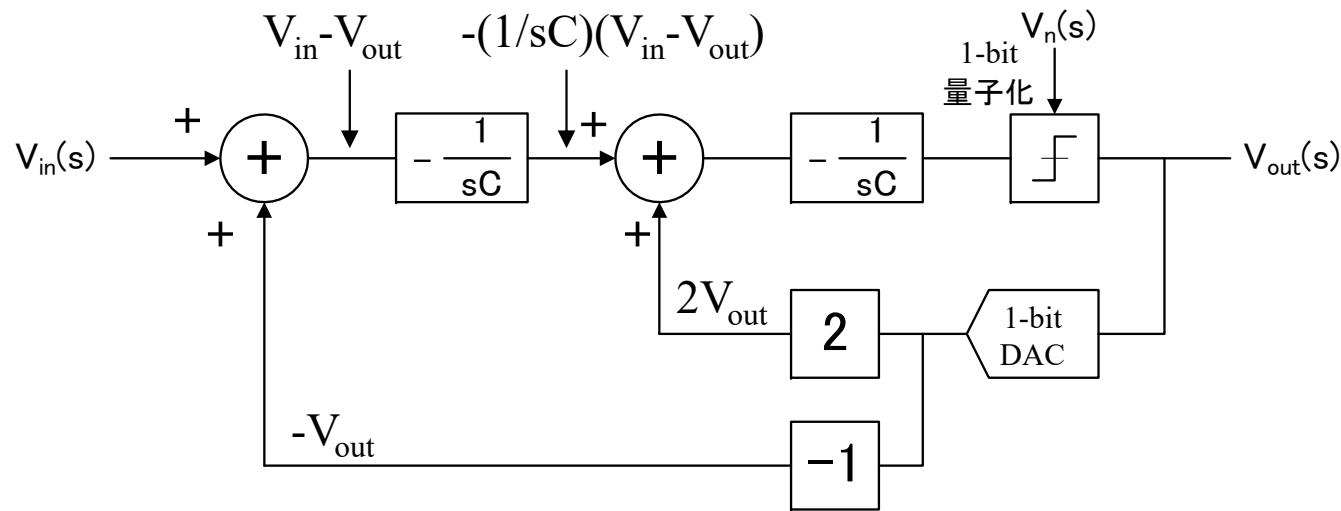


Problems 04

# **DELTA-SIGMA MODULATORS**

# Problem 1-1



$$V_{out} = -\frac{1}{sC} \left\{ -\frac{1}{sC} (V_{in} - V_{out}) + 2V_{out} \right\} + V_n$$

$$(1 + \frac{2}{sC} + \frac{1}{(sC)^2})V_{out} = \frac{1}{(sC)^2} V_{in} + V_n$$

$$V_{out} = \frac{1}{(sC+1)^2} V_{in} + \frac{(sC)^2}{(sC+1)^2} V_n$$

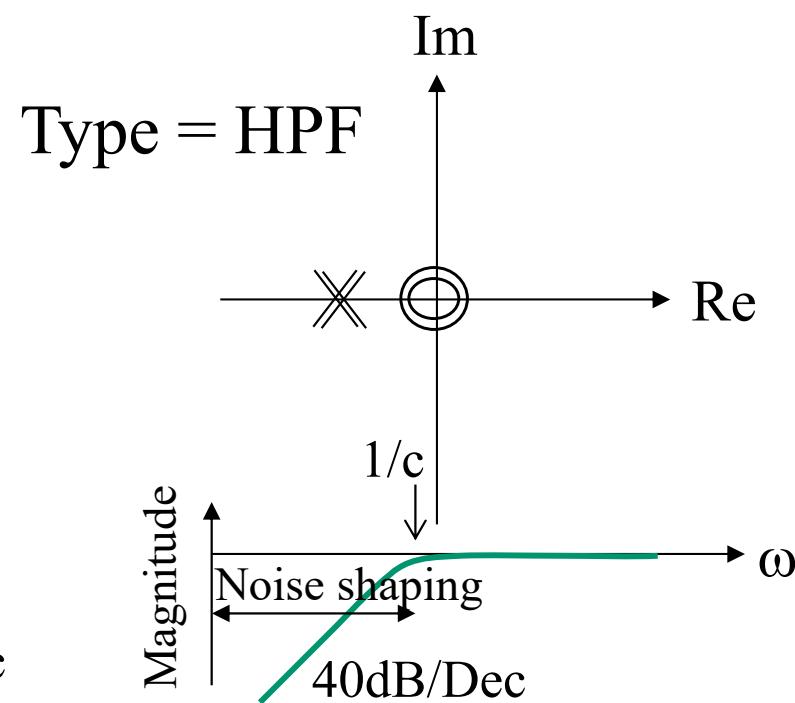
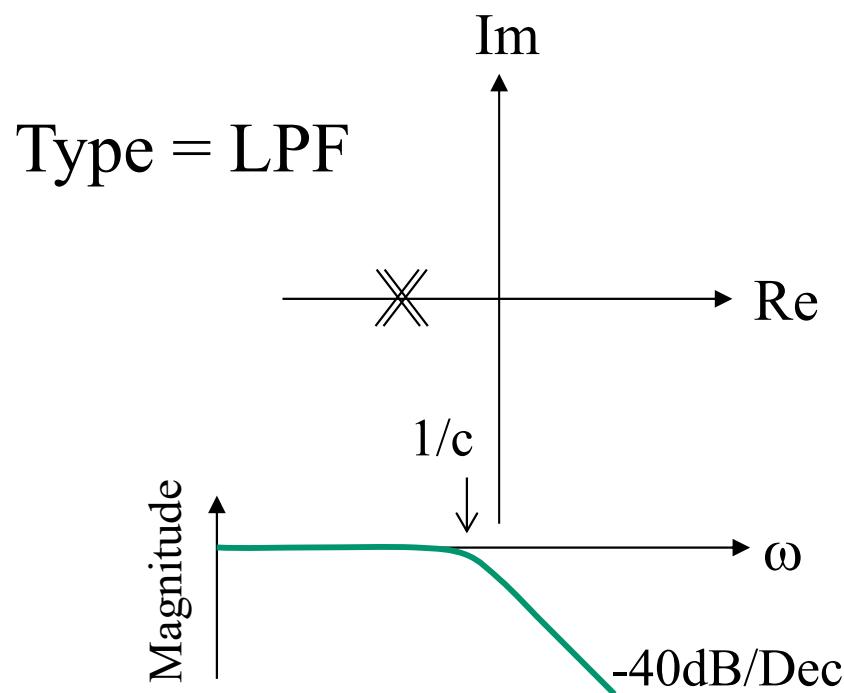
# Problem 1-2

$$H_s(s) = \frac{1}{(sC + 1)^2}$$

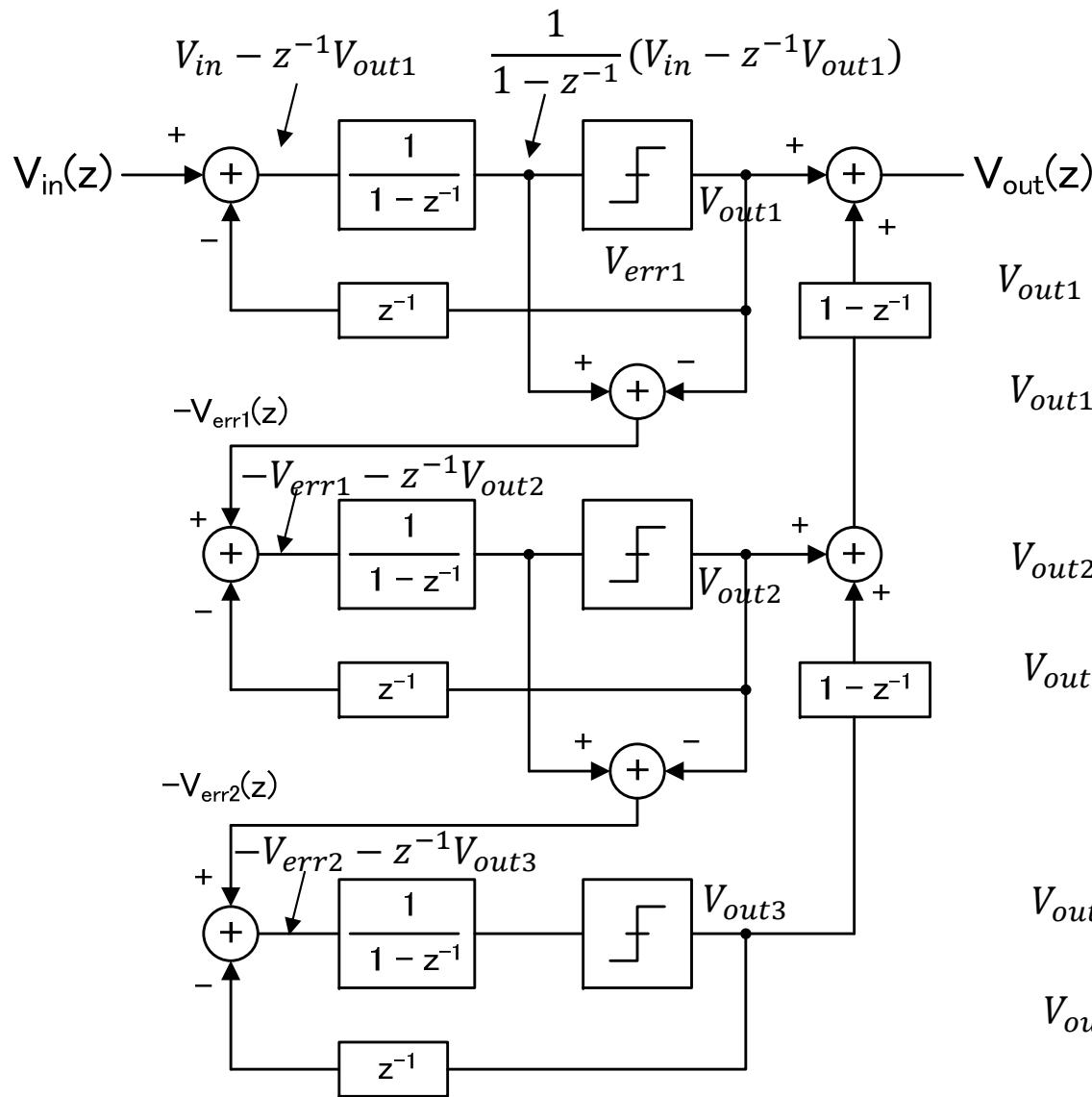
$$H_n(s) = \frac{(sC)^2}{(sC + 1)^2}$$

Zero	None
Pole	-1/C (Double)

Zero	0 (Double)
Pole	-1/C (Double)



# Problem 2-1



$$\frac{1}{1-z^{-1}}(V_{in} - z^{-1}V_{out1})$$

$$V_{out1} = \frac{1}{1-z^{-1}}(V_{in} - z^{-1}V_{out1}) + V_{err1}$$

$$V_{out1} = V_{in} + (1 - z^{-1})V_{err1}$$

$$V_{out2} = \frac{1}{1-z^{-1}}(-V_{err1} - z^{-1}V_{out2}) + V_{err2}$$

$$V_{out2} = -V_{err1} + (1 - z^{-1})V_{err2}$$

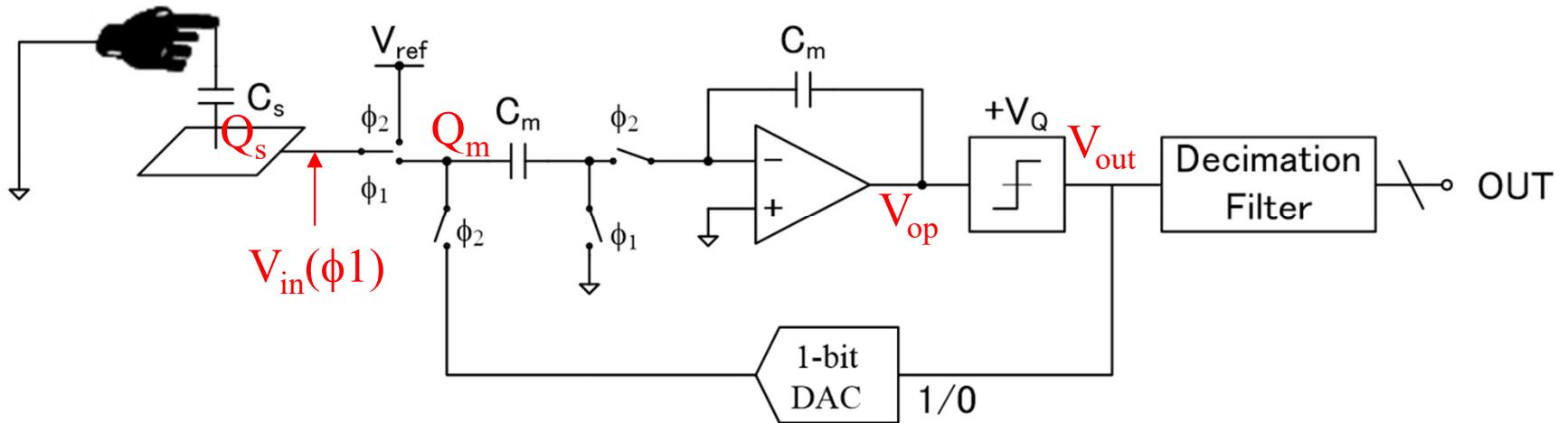
$$V_{out3} = \frac{1}{1-z^{-1}}(-V_{err2} - z^{-1}V_{out3}) + V_{err3}$$

$$V_{out3} = -V_{err2} + (1 - z^{-1})V_{err3}$$

# Problem 2-2

$$\begin{aligned}V_{out} &= V_{out1} + (1 - z^{-1})\{V_{out2} + (1 - z^{-1})V_{out3}\} \\&= V_{in} + (1 - z^{-1})V_{err1} + (1 - z^{-1})\{-V_{err1} + (1 - z^{-1})V_{err2} + (1 - z^{-1})\{-V_{err2} + (1 - z^{-1})V_{err3}\}\} \\&= V_{in} + (1 - z^{-1})V_{err1} - (1 - z^{-1})V_{err1} + (1 - z^{-1})^2V_{err2} - (1 - z^{-1})^2V_{err2} \\&\quad + (1 - z^{-1})^3V_{err3} \\&= V_{in} + (1 - z^{-1})^3V_{err3} \quad (\text{3rd order nose shaping})\end{aligned}$$

# Problem 3-1



In the phase  $\phi_2$ ,  $Q_s(t - \frac{T_s}{2}) = C_s V_{ref}$ ,  $Q_m(t - \frac{T_s}{2}) = C_m V_{out}(t - \frac{T_s}{2})$

In the phase  $\phi_1$ ,

$$V_{in}(t) = \frac{1}{C_s + C_m} \left\{ Q_s(t - \frac{T_s}{2}) = +Q_m \left( t - \frac{T_s}{2} \right) \right\}$$

$$= \frac{1}{C_s + C_m} \left\{ C_s V_{ref} + C_m V_{out}(t - \frac{T_s}{2}) \right\}$$

↓ Z

(The temporal changes in the capacitance  $C_s$  is slower than the sampling frequency. )

$$V_{in}(z) = \frac{1}{C_s + C_m} \left\{ C_s V_{ref} + z^{-\frac{1}{2}} C_m V_{out}(z) \right\}$$

# Problem 3-2

Using the equation shown in Chap.4, Slide 28,

$$\begin{aligned}
 V_{op}(z) &= \frac{C_m}{C_m} \frac{1}{1 - z^{-1}} \left\{ z^{-\frac{1}{2}} V_{in}(z) + V_{out}(z) \right\} \\
 &= \frac{1}{1 - z^{-1}} \left\{ \frac{z^{-\frac{1}{2}}}{C_s + C_m} \left\{ C_s V_{ref} + z^{-\frac{1}{2}} C_m V_{out}(z) \right\} - V_{out}(z) \right\} \\
 &= \frac{z^{-\frac{1}{2}}}{1 - z^{-1}} \frac{C_s}{C_s + C_m} V_{ref} + \frac{z^{-1}}{1 - z^{-1}} \frac{C_m}{C_s + C_m} V_{out}(z) - \frac{1}{1 - z^{-1}} V_{out}(z)
 \end{aligned}$$

$$\begin{aligned}
 V_{out}(z) &= z^{-1} V_{op}(z) + V_Q(z) \\
 &= \frac{z^{-\frac{3}{2}}}{1 - z^{-1}} \frac{C_s}{C_s + C_m} V_{ref} + \frac{z^{-2}}{1 - z^{-1}} \frac{C_m}{C_s + C_m} V_{out}(z) - \frac{z^{-1}}{1 - z^{-1}} V_{out}(z) + V_Q(z) \\
 &\approx \frac{z^{-\frac{3}{2}}}{1 - z^{-1}} \frac{C_s}{C_s + C_m} V_{ref} - \frac{z^{-1}}{1 - z^{-1}} V_{out}(z) + V_Q(z) \quad (C_s \gg C_m)
 \end{aligned}$$

$$V_{out}(z) \approx z^{-\frac{3}{2}} \frac{C_s}{C_s + C_m} V_{ref} + (1 - z^{-1}) V_Q(z)$$

# 問題3.2の補足説明

- この回路は、Capsenseと呼ばれ、タッチセンサや近接センサに使用される
- 参照入力付きDAIを使用しているが、入力が電圧ではなく、 $C_s$ に充電された電荷を用いるため、 $C_s + C_m$ の容量に入力電荷が分配されてしまう問題がある。前ページの計算では、 $C_s \gg C_m$  の近似を使用したが、この条件下では、 $C_s/(C_s + C_m) \approx 1$ となり、 $C_s$ の変化を検出する感度が小さくなる
- この問題を解決するために、入力用と帰還用のキャパシタを別々に用意する(第4章スライド28の中段の回路)