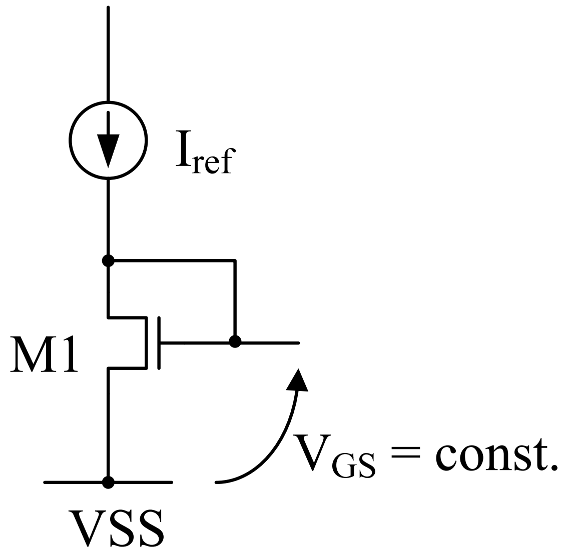
Summary of Chapter 8-11

Mixed signal LSI, Akio Kitagawa, Kanazawa University

1. Current mirror

Constant voltage circuit



$$\begin{cases} V_{DS} = V_{GS} \\ V_{DS} \geq V_{GS} - V_{Tn} = \Delta_{OV} \end{cases}$$

Therefore, M1 is driven in the saturation region. This circuit can output the voltage V_{GS} controlled by I_{ref}.

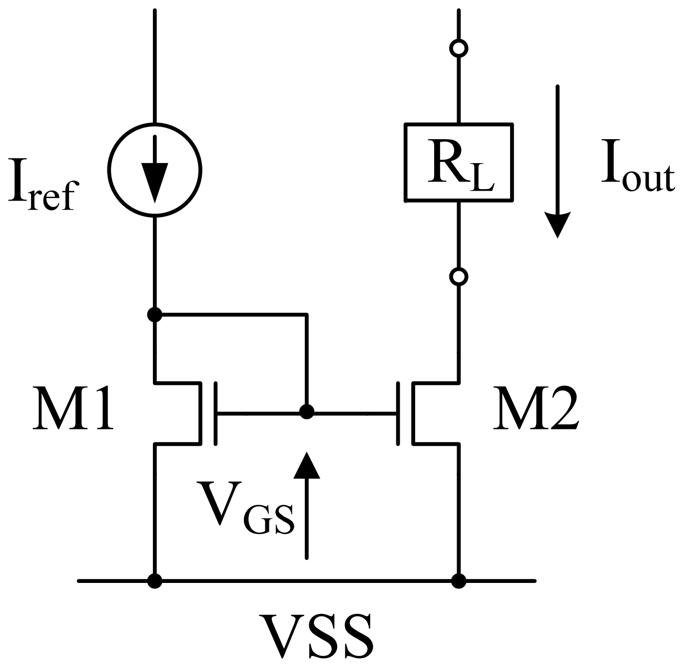
In the saturation region

I_D = const. → V_{GS} = const. V_{GS} = const. → I_D = const.

$$I_D = I_{ref} = \frac{\beta_1}{2}(V_{GS} - V_{Tn})^2$$

$$V_{GS}(I_{ref}) = V_{Tn} + \sqrt{\frac{2I_{ref}}{\beta_n}} = V_{Tn} + \Delta_{OV}$$ (Δ_{OV}: Overdrive voltage)

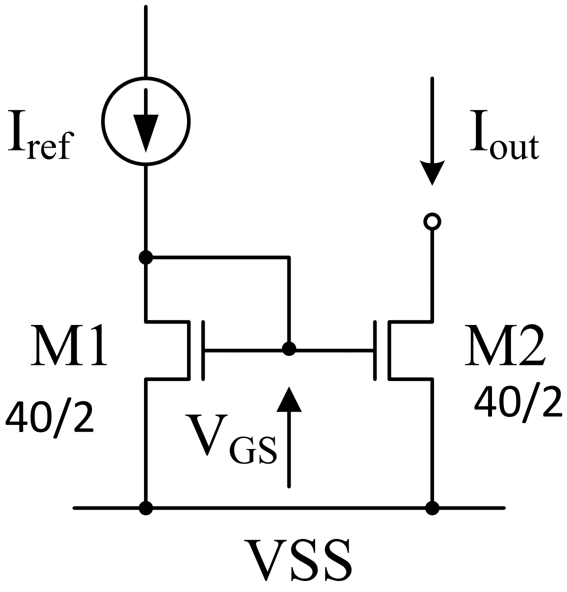
Current mirror

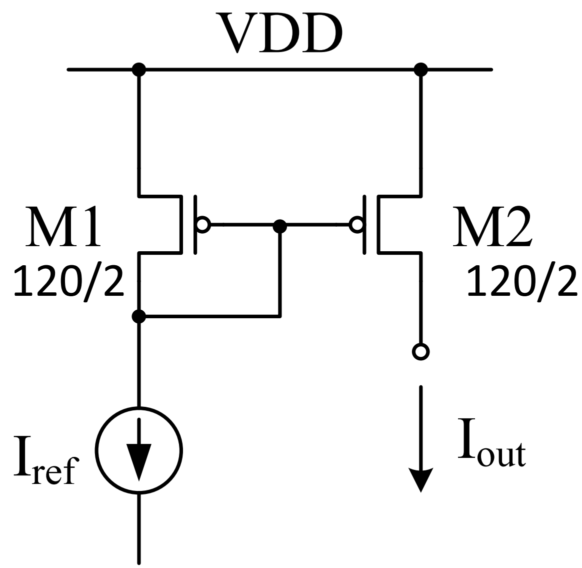


$$\begin{cases} I_{out} = \dfrac{\beta_2}{2}(V_{GS} - V_{Tn})^2 \\ I_{ref} = \dfrac{\beta_1}{2}(V_{GS} - V_{Tn})^2 \end{cases}$$

$$\therefore I_{out} = \frac{\beta_2}{\beta_1} I_{ref} = \frac{(\frac{W}{L})_2}{(\frac{W}{L})_1} I_{ref}$$

Sink and source of the current mirror



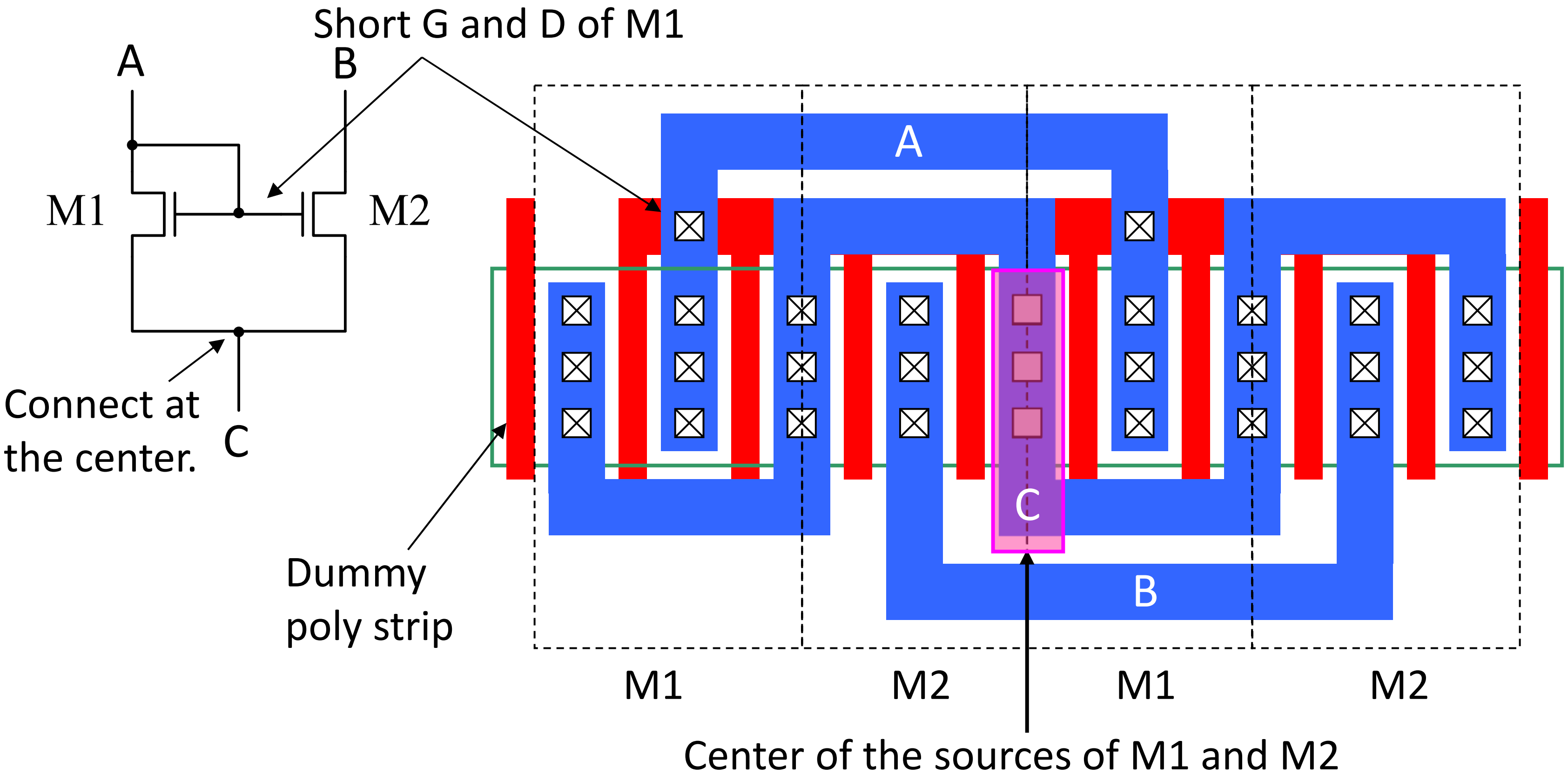


Current Sink

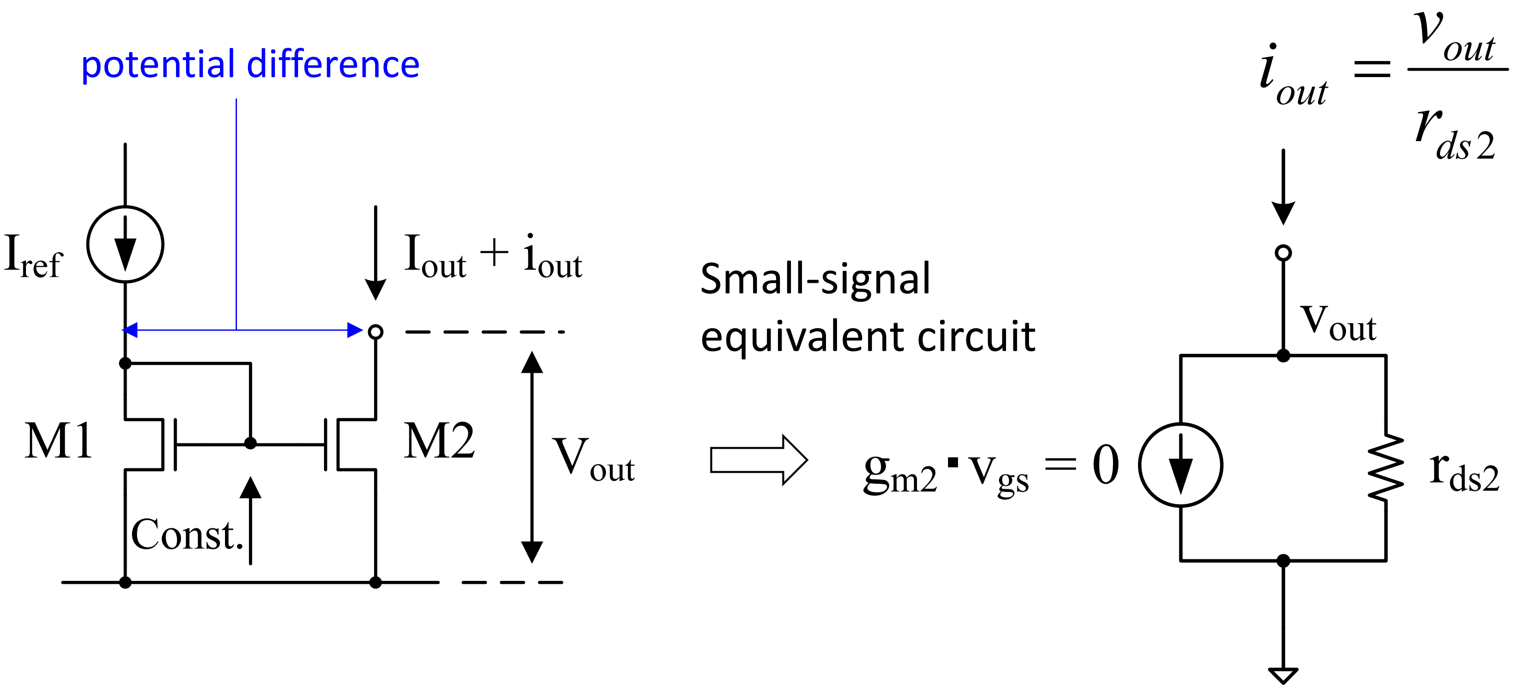
Current Source

$$I_{out} = \frac{(\frac{W}{L})_2}{(\frac{W}{L})_1} I_{ref}$$

Layout sample of the current mirror



Deviation from the ideal characteristics

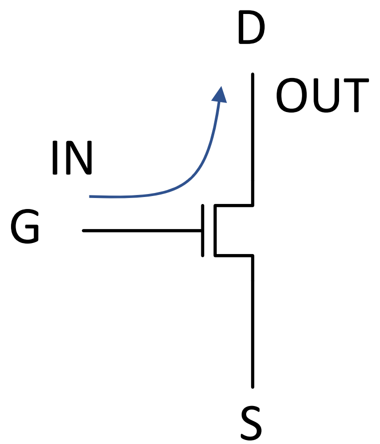


The channel resistance of M2 is not infinite, therefore, M2 cannot work as an ideal current source. The higher drain resistance of M2 is preferable to improve the characteristic of the current source.

2. Basic amplifier

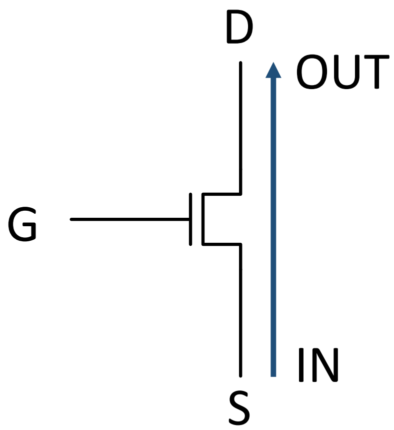
Signal transmission and common terminal

Common source



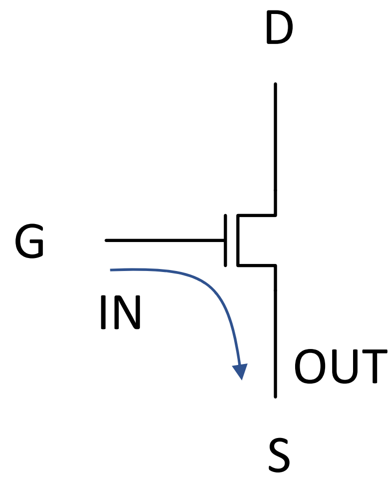
Inverting

Common gate



Non-inverting

Common drain



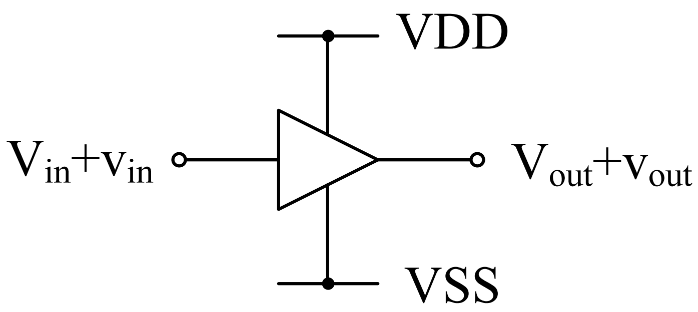
Non-inverting

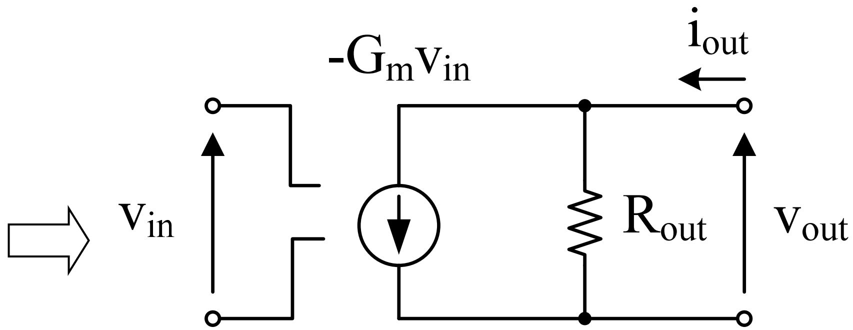
Configurations of a basic amplifiers

Types of amplifiers

	Common source	Common gate	Common drain
Source	Common	Input	Output
Gate	Input	Common	Input
Drain	Output	Output	Common
Function	Voltage Amplifier H→L impedance	Voltage Amplifier L→H trans-impedance	DC level shift H→L impedance

Voltage gain of amplifiers





Small-signal equivalent circuit (Behavior model)

Definition of G_m, R_{out}

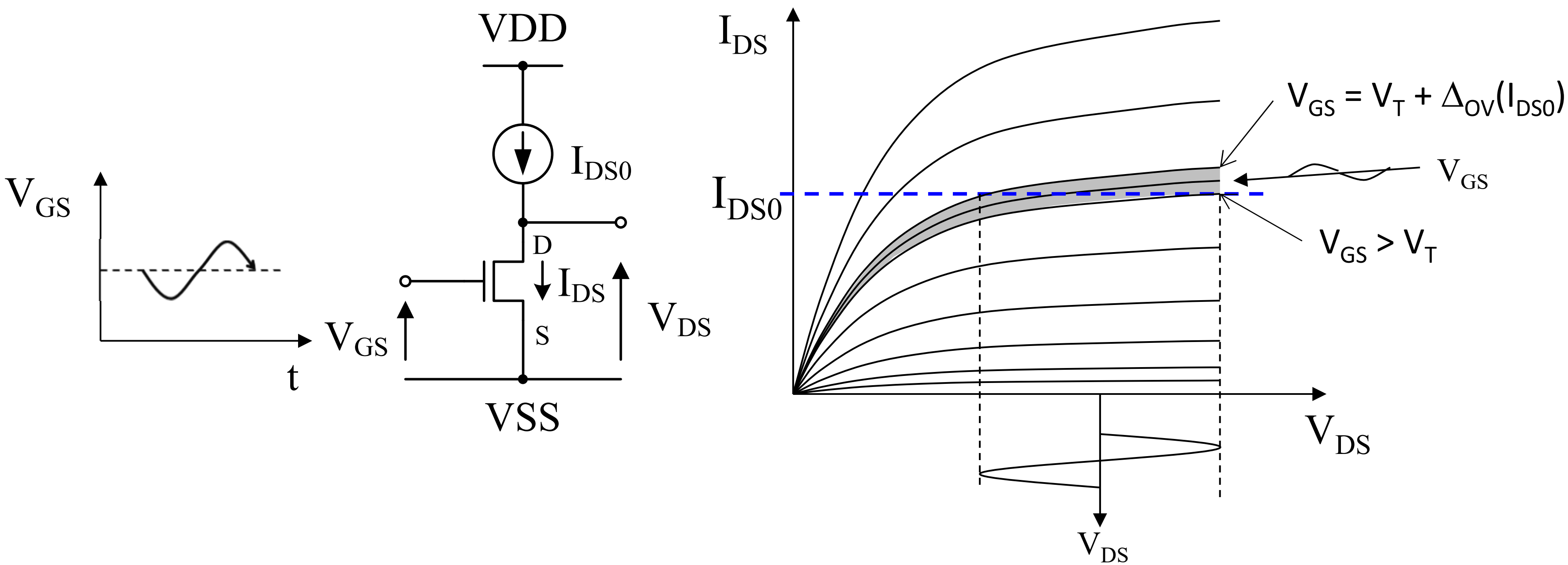
$$\begin{cases} G_m = -\dfrac{i_{out}}{v_{in}}\Big|_{v_{out}=0} \\[2ex] R_{out} = \dfrac{v_{out}}{i_{out}}\Big|_{v_{in}=0} \end{cases}$$

$$Av \equiv \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{-(-G_m \cdot v_{\text{in}}) \cdot R_{out}}{v_{\text{in}}} = G_m \cdot R_{out}$$

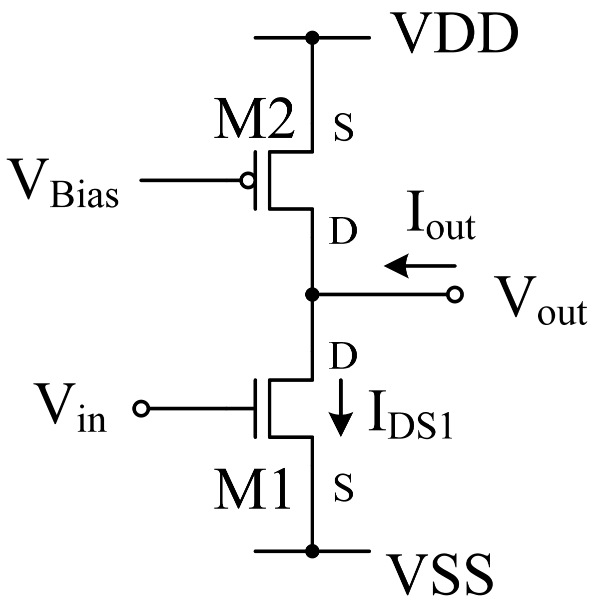
Assumption $\begin{cases} \text{Impedance of input signal} = 0 \\ \text{Impedance of output load} = \infty \end{cases}$

Common-source (CS) amplifier

Current source load I_{DS0} (MOSFET current source)
- Bias current I_{DS0} is supplied to M1
- Current source works as a large load resistance (Large voltage gain)



Voltage gain of CS amplifier



$$G_m \equiv -\frac{i_{out}}{v_{in}}\bigg|_{v_{out}=0} = -\frac{i_{ds1}}{v_{gs1}} = -g_{m1}$$

$$R_{out} \equiv \frac{v_{out}}{i_{out}}\bigg|_{v_{in}=0} = \frac{1}{g_{ds1}+g_{ds2}} = r_{ds1}//r_{ds2}$$

Please draw a small signal equivalent circuit for yourself.

$$A_v \equiv G_m \cdot R_{out} = \frac{-g_{m1}}{g_{ds1}+g_{ds2}} = -g_{m1}\cdot(r_{ds1}//r_{ds2}) = -g_{m1}\frac{r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}$$

Setting of the bias current (1)

I-V characteristic in saturation region ($V_{DS} > V_{GS}$ - V_T)

$$I_{DS} = \frac{1}{2}\frac{W}{L}\mu_n \cdot C_{OX}(V_{GS} - V_T)^2\{1 + \lambda(V_{DS} - \Delta_{OV})\}$$

$$= \frac{\beta}{2}(V_{GS} - V_T)^2\{1 + \lambda(V_{DS} - \Delta_{OV})\}$$

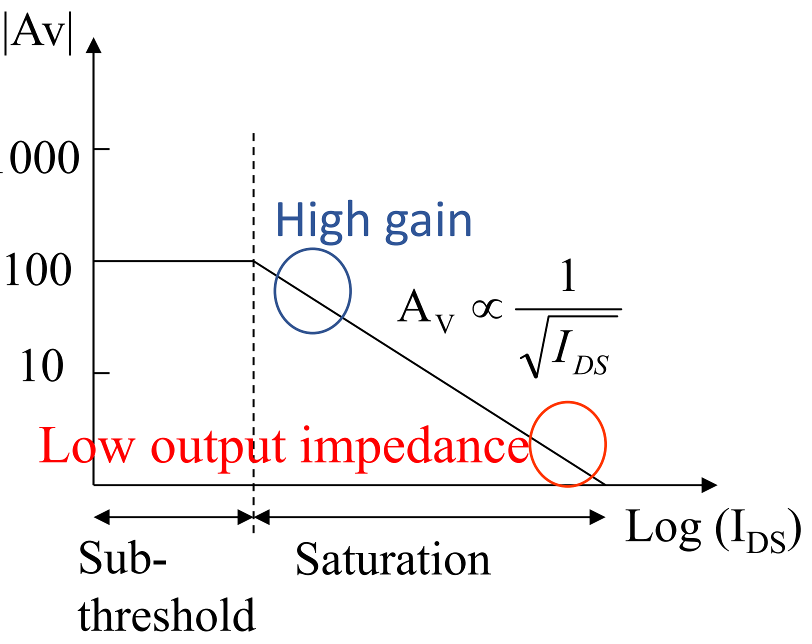
$$\Downarrow$$

$$g_m \equiv \frac{\partial I_{DS}}{\partial V_{GS}} \approx \beta(V_{GS} - V_T) = \sqrt{2\beta \cdot I_{DS}}$$

$$g_{ds} \equiv \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\beta}{2}(V_{GS} - V_T)^2 \lambda \approx \lambda \cdot I_{DS}$$

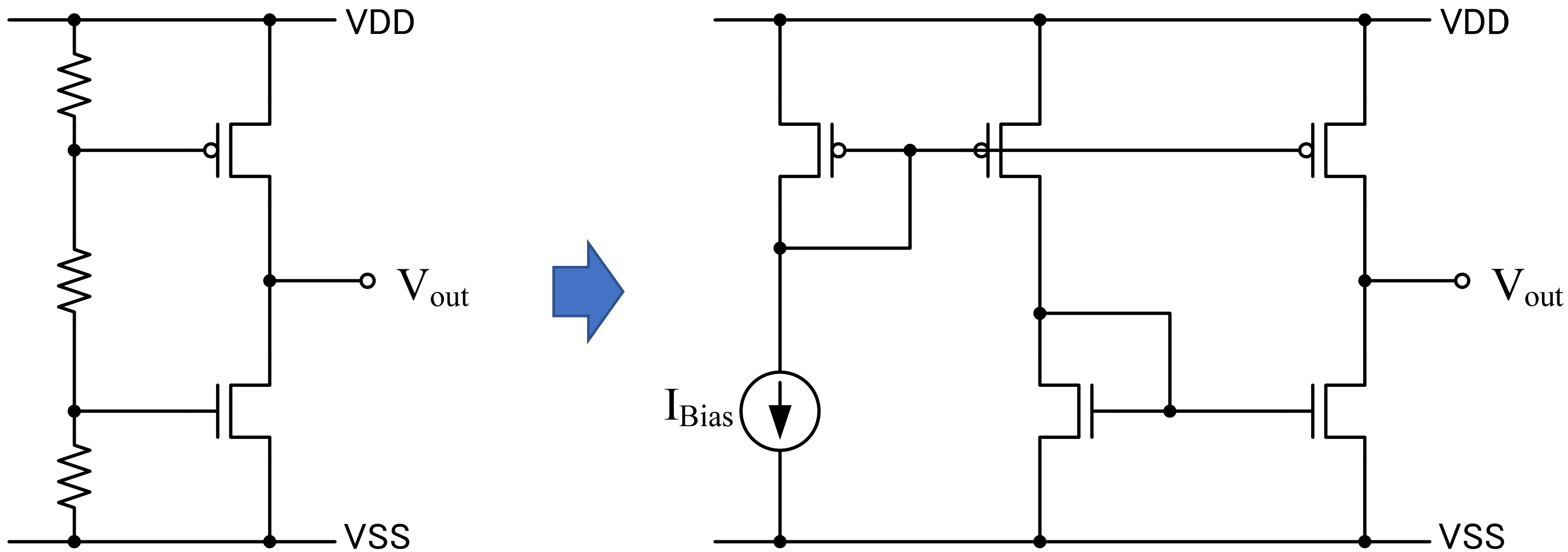
Setting of the bias current (2)

$$
\begin{aligned}
A_V &= -\frac{g_{m1}}{g_{ds1} + g_{ds2}} \\
&= -\frac{\sqrt{2\beta_1 I_{DS}}}{\lambda_1 I_{DS} + \lambda_2 I_{DS}} \\
&= -\frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2}\frac{1}{\sqrt{I_{DS}}}
\end{aligned}
$$



NOTE: gm/I_{DS} is a reference index of the voltage gain in saturation region.

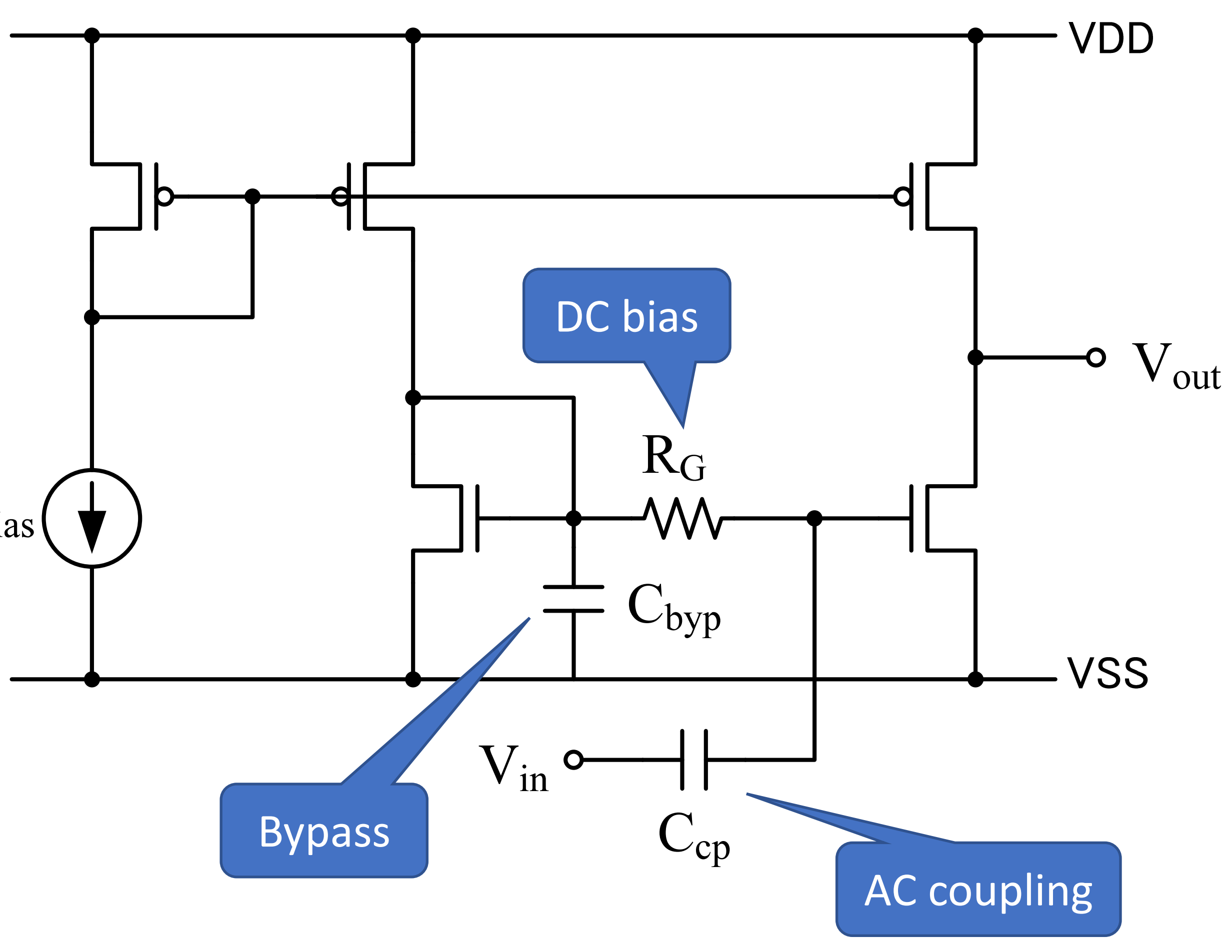
Bias circuit for CS amplifier



Voltage bias
(Unstable)

Current bias
(Stable)

AC coupling of CS amplifier



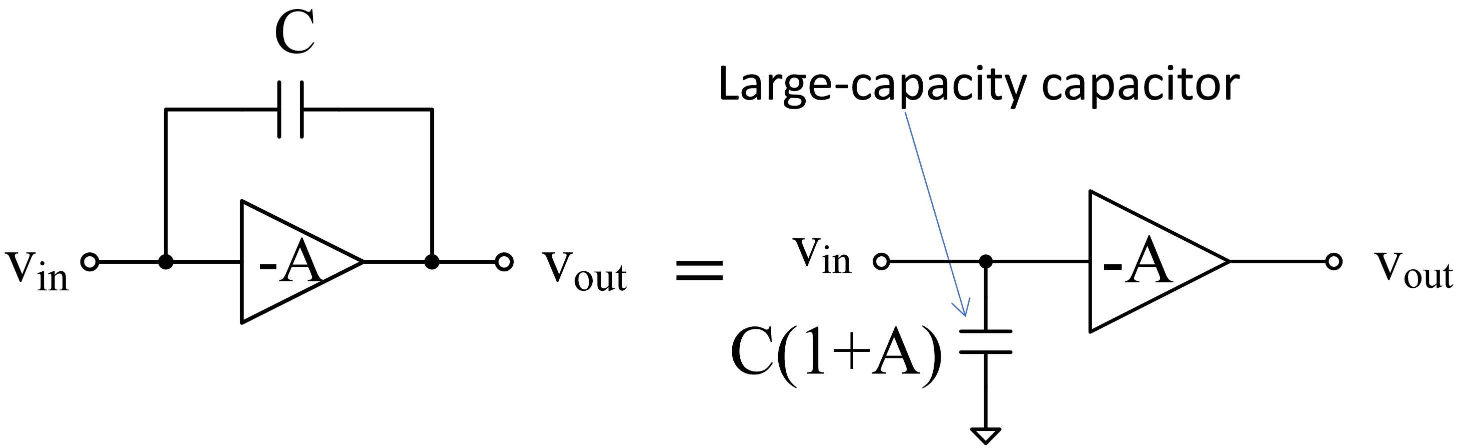
2. Cascode amplifier

Gain enhancement technique

- $$A_V = G_m \cdot R_{out} = -\frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2} \frac{1}{\sqrt{I_{DS}}}$$ (CS amplifier)
- The high voltage gain is achieved by the large β_1 or small I_{DS}, but the small I_{DS} or small Δ_{OV} increases influence of the process variation and the large β_1 or large W causes a Miller effect.
- The cascode technique is useful for the gain enhancement by increasing the output resistance of amplifiers without additional bias current.

Miller effect

The AC characteristic of amplifiers is remarkably degraded by the Miller effect.



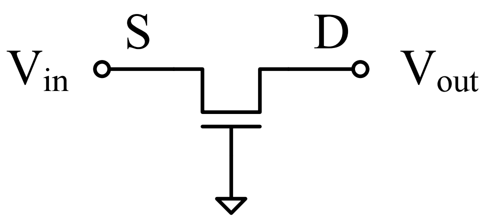
$$\begin{cases} v_{out} = -A \cdot v_{in} \\ i_{in} = j\omega \cdot C(v_{in} - v_{out}) = j\omega \cdot C \cdot (A+1) \cdot v_{in} \end{cases}$$

Equivalent circuit

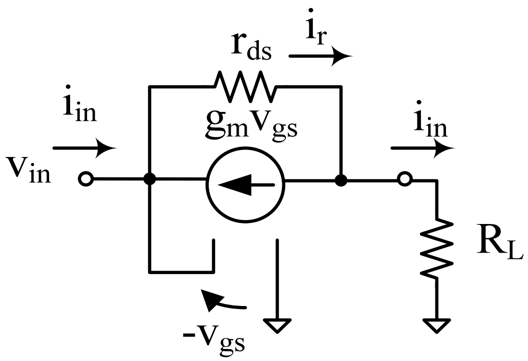
$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{j\omega \cdot C \cdot (A+1)}$$

Assuming that the input impedance is very large.

Trans-impedance with CG amplifier



Voltage gain of CG amplifier: $A_V = 1 + g_m \cdot r_{ds}$

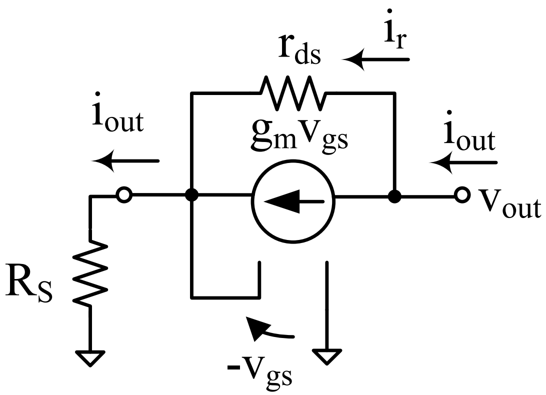


Input Impedance

$$\begin{cases} v_{in} = -v_{gs} \\ i_{in} + g_m v_{gs} = i_r \\ v_{in} = r_{ds} i_r + R_L i \end{cases}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_{ds} + R_L}{1 + g_m r_{ds}} = \frac{r_{ds} + R_L}{A_V} \approx \underbrace{\frac{1}{g_m}} \quad (\text{if } R_L = 0)$$

Memorize



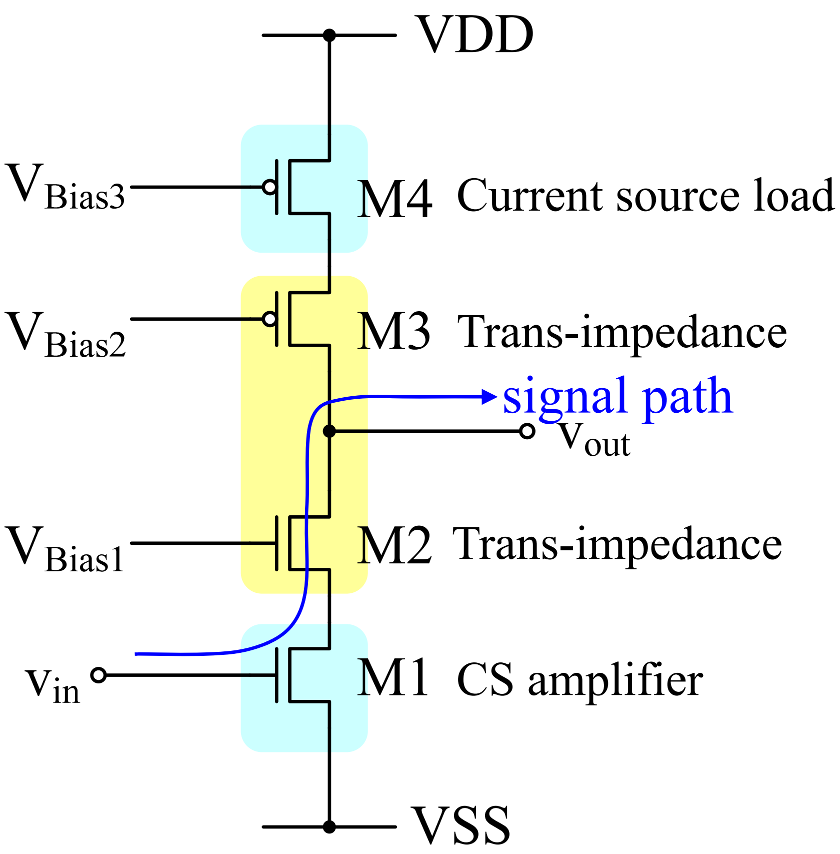
Output Impedance

$$\begin{cases} -v_{gs} = R_S i_{out} \\ i_{out} = g_m v_{gs} + i_r \\ v_{out} = r_{ds} i_r + R_S i_{out} \end{cases}$$

$$R_{out} = \frac{v_{out}}{i_{out}} = r_{ds}(1 + g_m R_S) + R_S \approx \underbrace{A_V R_S}$$

Memorize

Cascode amplifier



$$A_V = G_m R_{out}$$
$$G_m = -g_{m1}$$
$$R_{out} = \{(g_{m2} r_{ds2}) r_{ds1}\} // \{(g_{m3} r_{ds3}) r_{ds4}\}$$

If $r_{ds1} = r_{ds4}, r_{ds2} = r_{ds3}, g_{m2} = g_{m3},$

$$A_V = -\frac{1}{2} g_{m1} r_{ds1} g_{m2} r_{ds2}$$

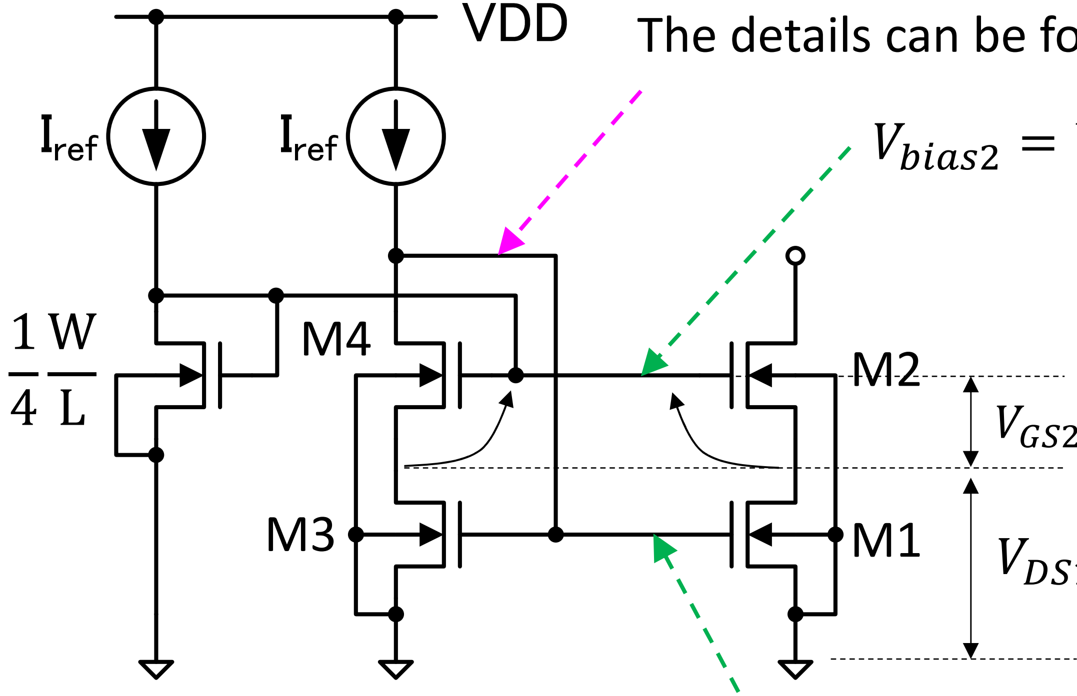
Voltage gain of CS amp. ~30dB ($=g_{m1} \cdot r_{ds1}$)

Voltage gain of cascode amp. > 60dB without additional power consumption.

However, the cascode circuit designed for low VDD is disadvantageous for the output swing, because the stacked MOFETs should be driven in the saturation region.

Cascode current mirror

Technique to achieve $V_{DS3} = V_{OV}$
The details can be found in chapter 8.



$$V_{bias2} = V_{Tn} + 2V_{OV}$$

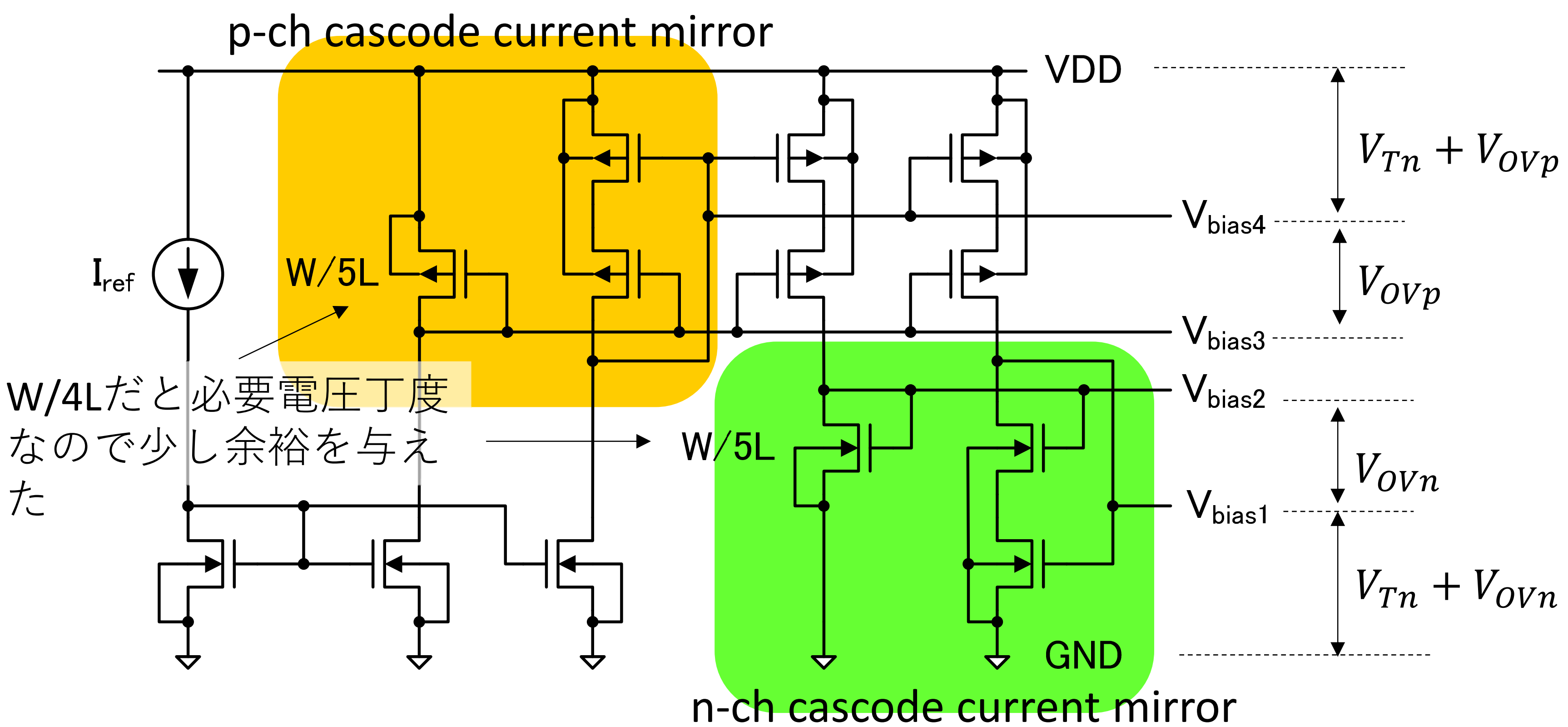
$$V_{OV} = \sqrt{\frac{2I_{ref}}{\beta_n}}$$

$$V_{GS2} = V_{GS4} = V_{Tn} + V_{OV}$$

$$V_{DS1} = V_{DS3} = V_{OV} \text{ (Minimum)}$$

$$V_{bias1} = V_{Tn} + V_{OV}$$

Bias circuit for cascode amplifier



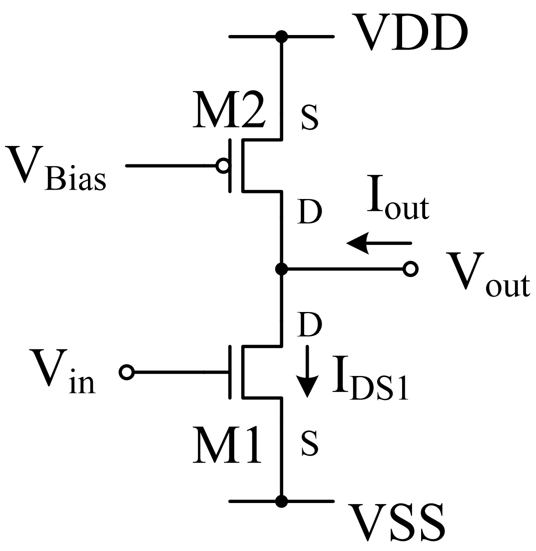
3. AC performance of amplifiers

AC characteristics of CS amplifier

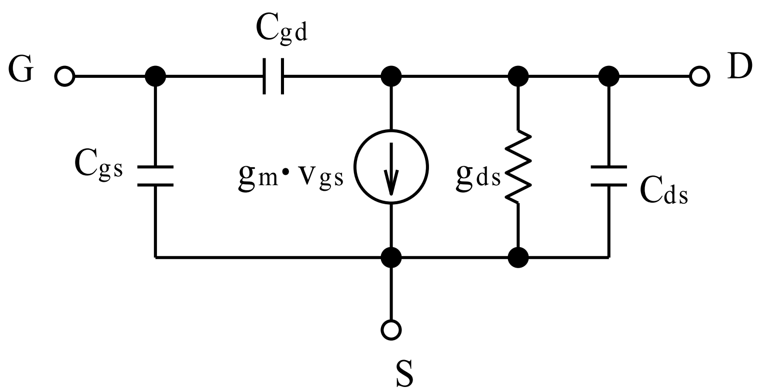
- A main factor to decide an AC characteristic
  - Output capacitance
    - Capacitive Load + Parasitic capacitance
  - Input capacitance
    - Parasitic capacitance
  - Input-output capacitance
    - Parasitic capacitance

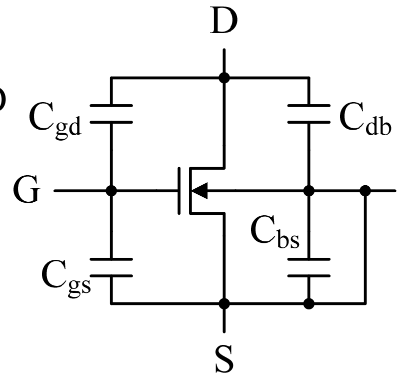
(AC characteristic: The small-signal frequency response)

Parasitic capacitance

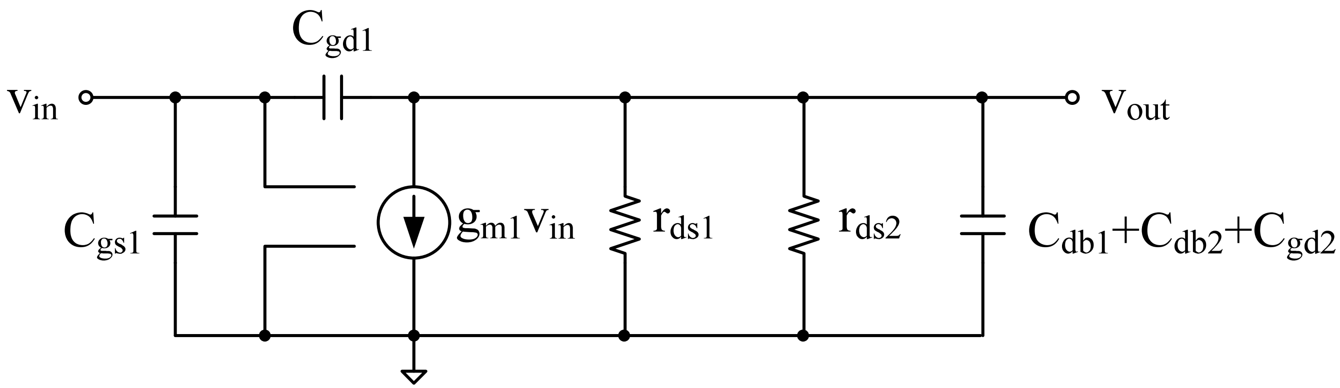


High frequency small-signal equivalent circuit of MOSFET

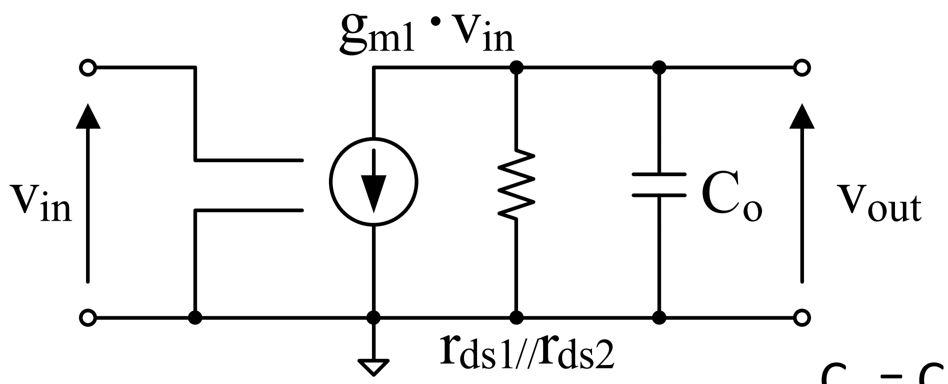




High frequency small-signal equivalent circuit of CS amplifier



Influence of the output capacitance



$C_o = C_{db1} + C_{db2} + C_{gd2} + C_L$

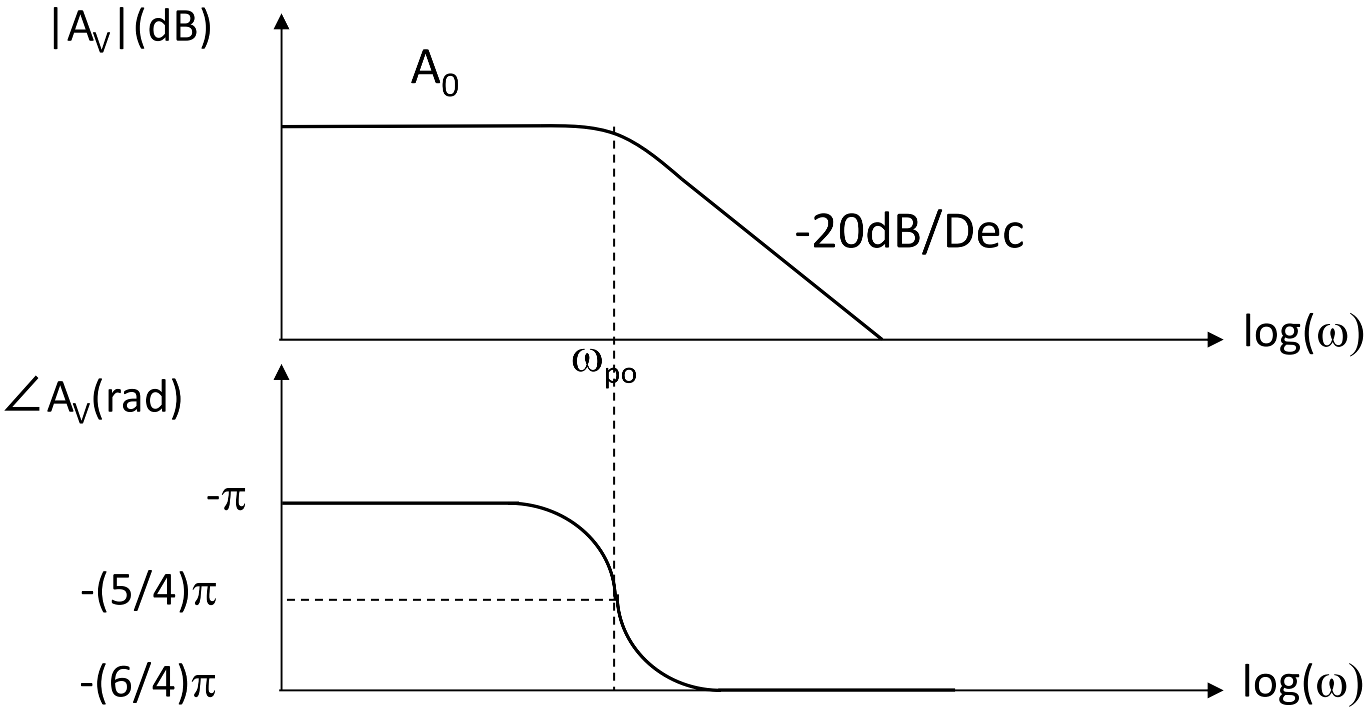
$$v_{out} = \frac{1}{\frac{1}{r_{ds1} // r_{ds2}} + j\omega \cdot C_O} (-g_m v_{in})$$

$$A(\omega) \equiv \frac{v_{out}}{v_{in}} = \frac{-g_m (r_{ds1} // r_{ds2})}{1 + j\omega \cdot C_O (r_{ds1} // r_{ds2})}$$

$$= \frac{-A_0(\omega = 0)}{1 + j\omega \cdot C_O (r_{ds1} // r_{ds2})}$$

Bode diagram of the CS amplifier

$$|A(\omega)| = \frac{A_0}{\sqrt{1+\omega^2/\omega_{po}^2}} \qquad \omega_{po} = \frac{1}{C_o(r_{ds1}//r_{ds2})}$$ (pole frequency of output)



Bias dependence of a pole frequency

$$\begin{cases} g_{m1} = \sqrt{2\beta_1 I_{DS1}} \\ r_{ds1} // r_{ds2} = \dfrac{1}{g_{ds1} + g_{ds2}} = \dfrac{1}{(\lambda_1 + \lambda_2) \cdot I_{DS1}} \end{cases}$$

$$A_0 = g_{m1}(r_{ds1} // r_{ds2}) = \frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2} \frac{1}{\sqrt{I_{DS1}}}$$ (DC gain)

$$\omega_{po} = \frac{1}{C_o(r_{ds1} // r_{ds2})} = \frac{(\lambda_1 + \lambda_2) \cdot I_{DS1}}{C_o}$$ (pole frequency)

$$\omega_{po} \cdot A_0^2 = \frac{1}{C_o} \frac{2\beta}{(\lambda_1 + \lambda_2)}$$

The product of the ω_{po} and A_0^2 is independent on the bias current.

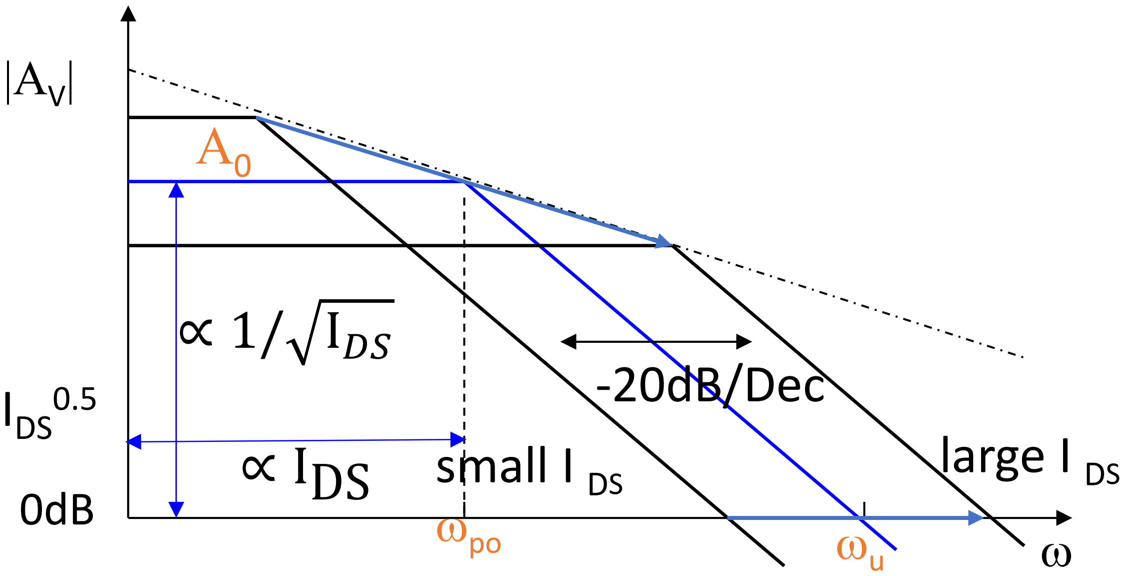
Unity gain frequency ω_u (= BGP)

$$\frac{A_0}{\sqrt{1+\omega_u^2/\omega_{po}^2}} = 1$$

$$\omega_u = \sqrt{(A_0^2-1)}\,\omega_{po}$$

$$\cong A_0\omega_{po} = \frac{g_m}{C_o} \quad \propto I_{DS}^{0.5}$$

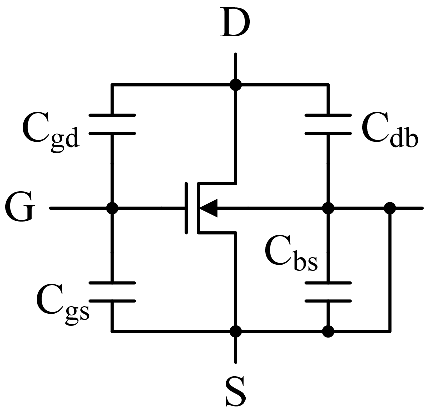
$$= \frac{\beta(V_{GS}-V_T)}{C_o}$$

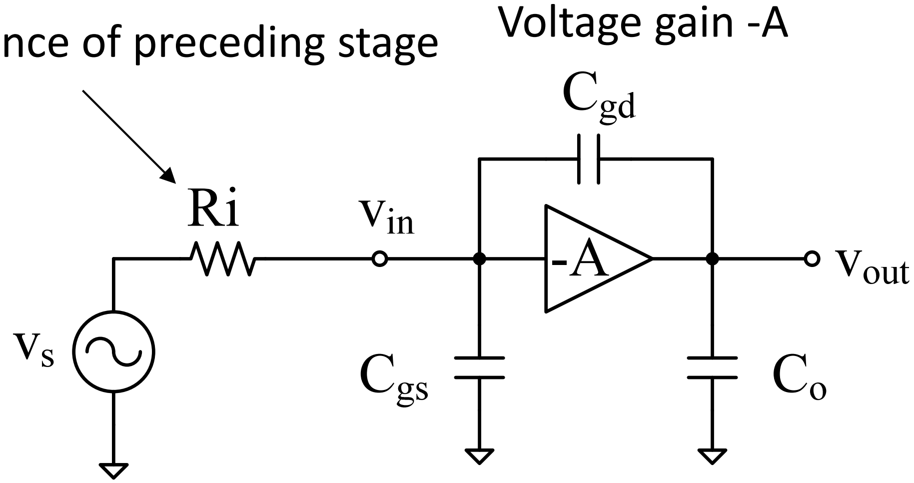


I_D ↑ : A_0↓, ω_{po}↑, ω_u↑

NOTE: ω_u ≒ GBP (Gain Bandwidth Product)

Influence of the input capacitance





Miller effect

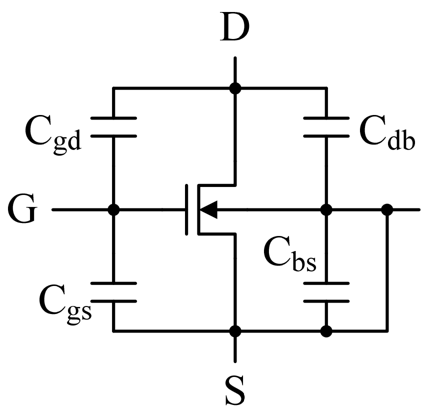
$$C_i = C_{gs1} + C_{gd1}(1 + A) \cong AC_{gd1}$$

$C_O = C_{db1} + C_{db2}\ \ C_{gd2} + C_L$

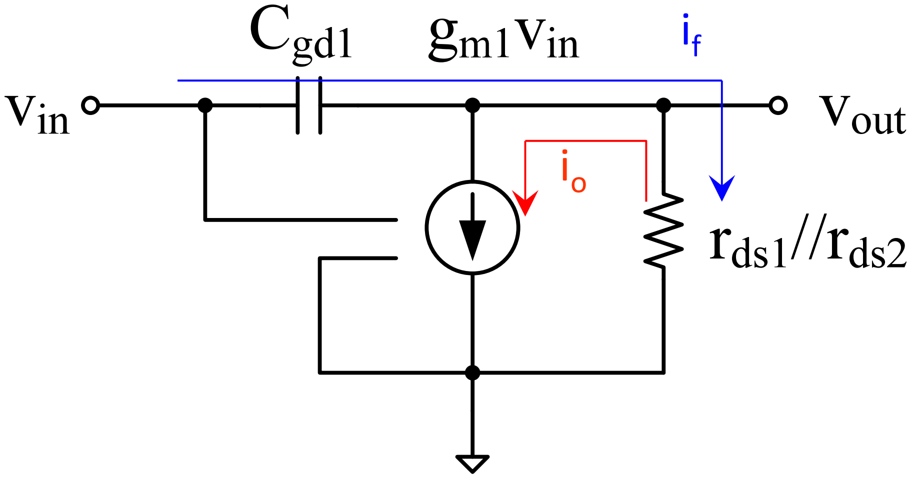
$$\omega_{pi} \cong \frac{1}{AR_iC_{gd1}}$$ (pole frequency of input)

NOTE: The pole frequency is low when a voltage gain is large.

Influence of the input-output capacitance (1)



Current path through C_{gd}



$$\begin{cases} v_{out} = (r_{ds1} // r_{ds2})(i_f - i_o) \\ i_f = j\omega \cdot C_{gd}(v_{in} - v_{out}) \end{cases}$$

Influence of the input-output capacitance (1)

by competitive current i_o and i_f

Normally $(r_{ds1}//r_{ds2}) < 1/g_{m1}$,
$\omega_z < \omega_{pgd}$

$$\frac{v_{out}}{v_{in}} = \frac{-(r_{ds1}//r_{ds2})g_{m1}(1-j\omega\cdot\frac{C_{gd}}{g_{m1}})}{1+j\omega\cdot C_{gd}(r_{ds1}//r_{ds2})} \equiv \frac{-(r_{ds1}//r_{ds2})g_{m1}(1-j\omega/\omega_z)}{1+j\omega/\omega_{pgd}}$$

i_f : Forward transmission signal from G to D
i_o : Normally amplified signal

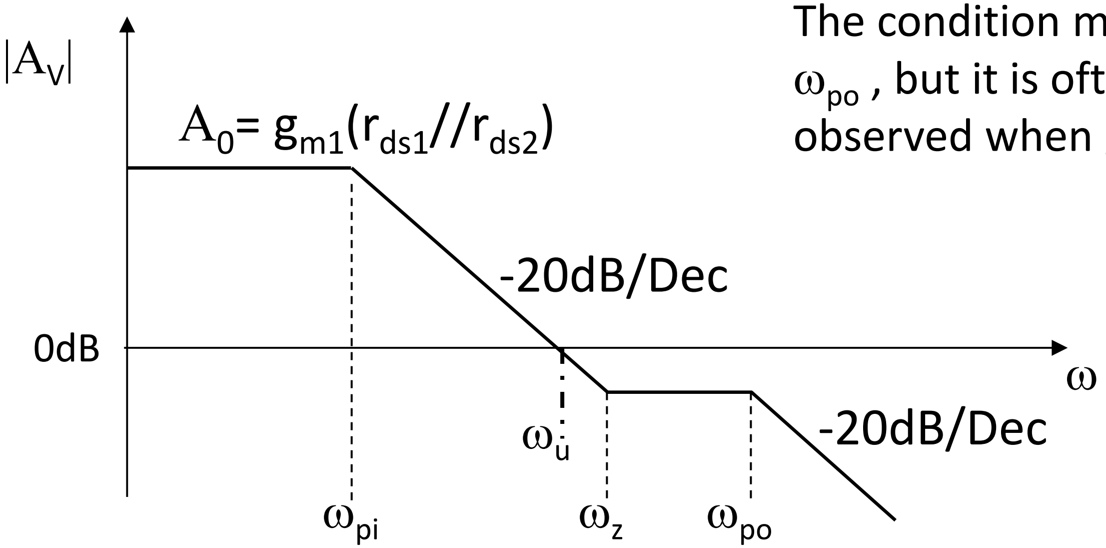
The balance of i_f and i_o generate the zero.

Summary of AC characteristics of the CS amplifier

$$A_V = \frac{A_0(1 - j\omega/\omega_z)}{(1 + j\omega/\omega_{pi})(1 + j\omega/\omega_{po})}$$

2 -pole and 1-zero transfer function

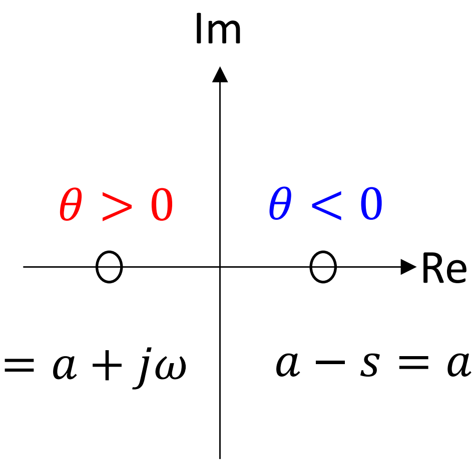
(The ω_{pgd} is placed in the very high frequency, thus it is usually negligible.)



The condition may not keep $\omega_{pi} < \omega_z < \omega_{po}$, but it is often that this condition is observed when g_{m1} is not very large.

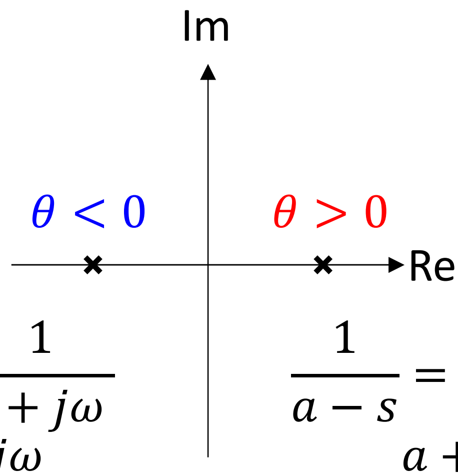
NOTE: Pole in the left half plane and zero in the right half plane turns phase -90 degrees.

Phase characteristic



$a + s = a + j\omega$

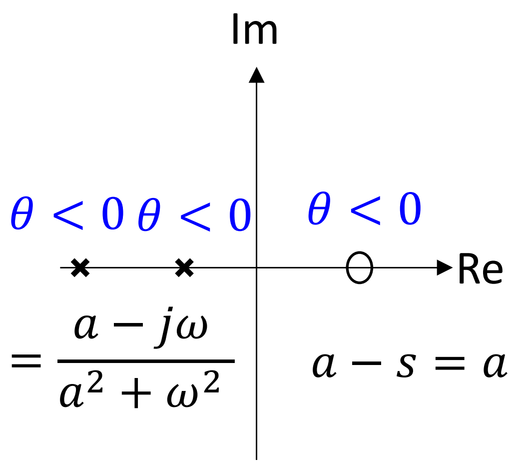
$a - s = a - j\omega$



$$\frac{1}{a+s} = \frac{1}{a+j\omega} = \frac{a-j\omega}{(a+j\omega)(a-j\omega)}$$

$$\frac{1}{a-s} = \frac{1}{a-j\omega} = \frac{a+j\omega}{(a-j\omega)(a+j\omega)}$$

CS amplifier



$$\frac{1}{a+s} = \frac{a-j\omega}{a^2+\omega^2}$$

$a - s = a - j\omega$

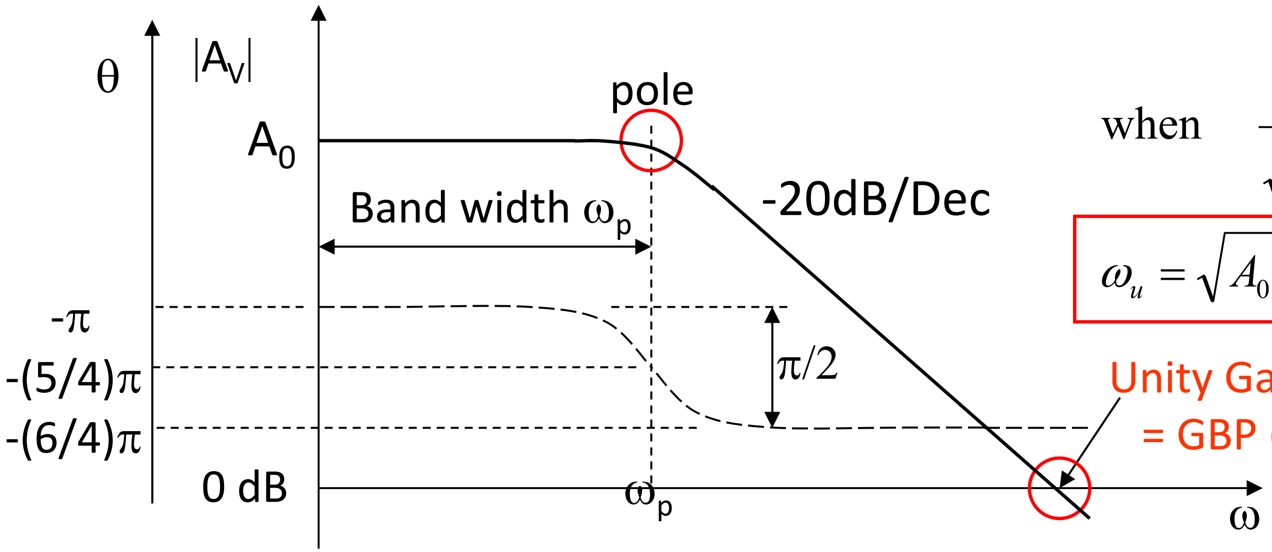
All pole and zero retard the negative phase of transfer function.

Simplified 1-pole model of AC characteristic of amplifiers

$$A_V(\omega) = \frac{A_0}{1 + j\omega/\omega_p} = \frac{A_0 \cdot \omega_p}{j\omega + \omega_p} \cong \frac{\omega_u}{j\omega + \omega_p}$$

$$|A_V(\omega)| = \left|\frac{v_{out}}{v_{in}}\right| = \frac{A_0}{\sqrt{1 + \omega^2/\omega_p^2}}$$

ω_p: the pole frequency (or cut-off frequency)
ω_u: the unity gain frequency



when $\dfrac{A_0}{\sqrt{1 + \omega_u^{\ 2}/\omega_p^2}} = 1$

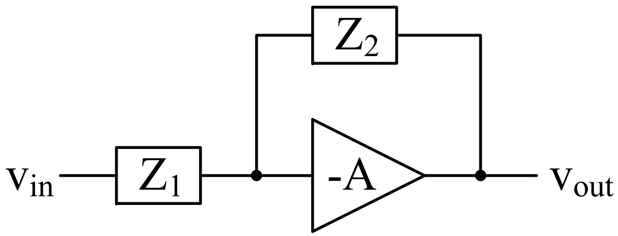
Important

$$\boxed{\omega_u = \sqrt{A_0^{\ 2} - 1} \cdot \omega_{p1} \cong A_0 \cdot \omega_p \equiv GBP}$$

Unity Gain Frequency ω_u
= GBP (Gain-bandwidth product)

Gain error of circuits and GBP

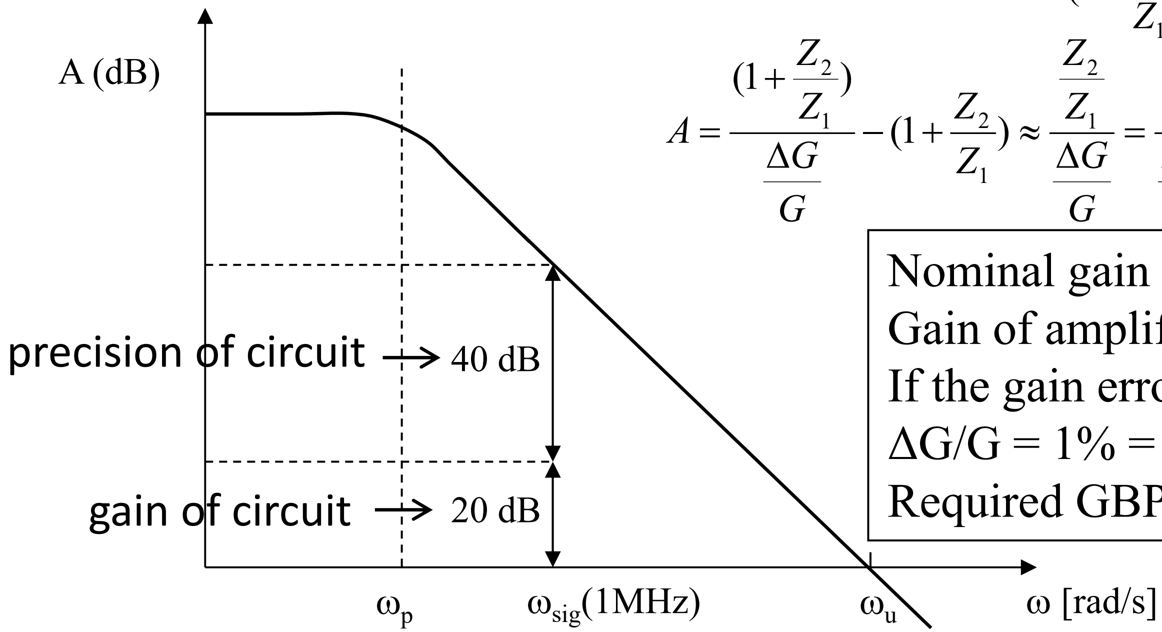
Gain of NFB amplifier
$$G = \frac{v_{out}}{v_{in}} = \frac{-\dfrac{Z_2}{Z_1}}{1+\dfrac{1+\dfrac{Z_2}{Z_1}}{A}} \xrightarrow{A\to\infty} -\frac{Z_2}{Z_1}$$



NFB amplifier

Gain error of NFB amplifier
$$\frac{\Delta G}{G} = \frac{G(\infty)-G(A)}{G(\infty)} = \frac{1+\dfrac{Z_2}{Z_1}}{A+(1+\dfrac{Z_2}{Z_1})}$$

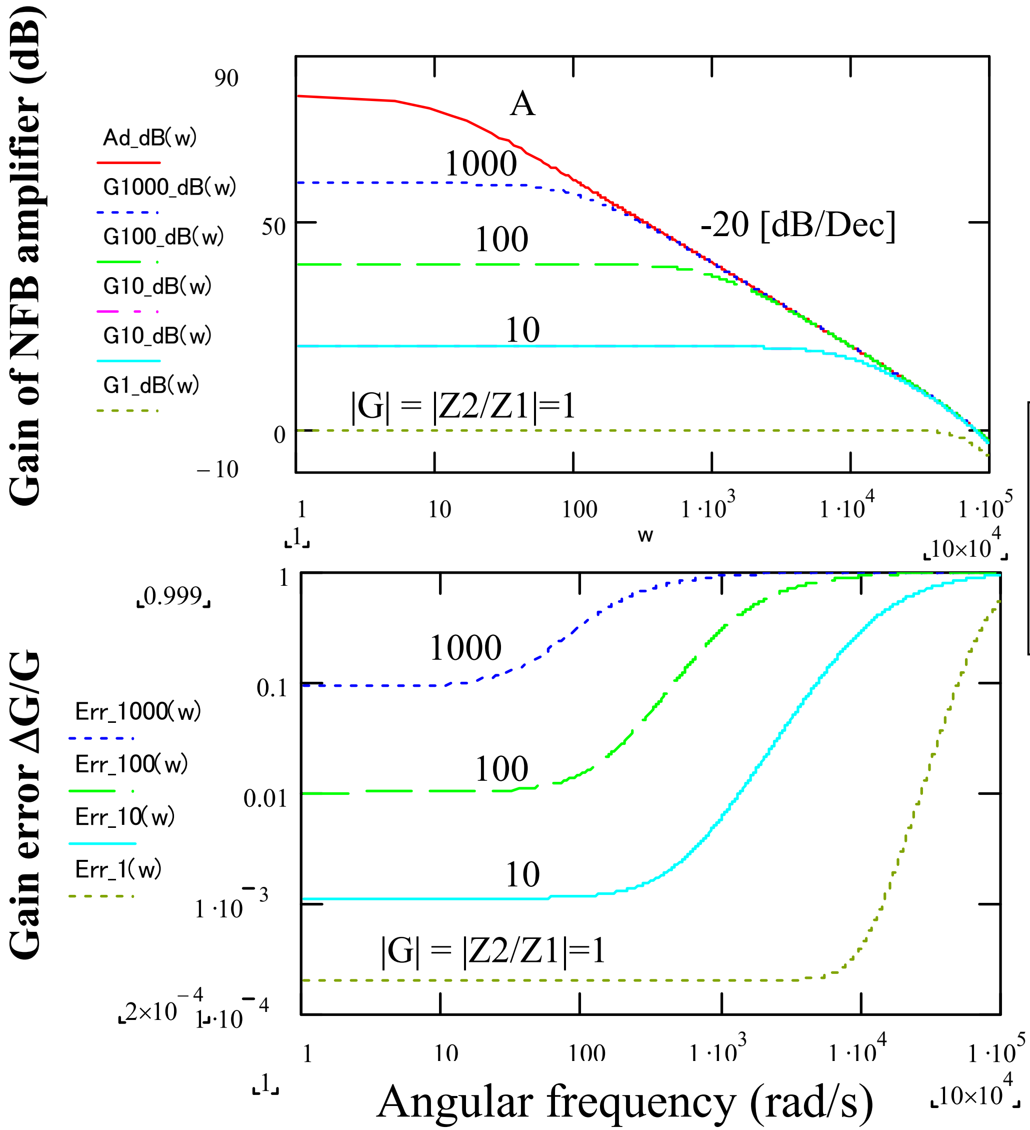
$$A = \frac{(1+\dfrac{Z_2}{Z_1})}{\dfrac{\Delta G}{G}} - (1+\frac{Z_2}{Z_1}) \approx \frac{\dfrac{Z_2}{Z_1}}{\dfrac{\Delta G}{G}} = \frac{G}{\dfrac{\Delta G}{G}} = G[dB] - \frac{\Delta G}{G}[dB]$$



Nominal gain G = 10.0 = 20 [dB] (Ideal value)
Gain of amplifier A at ω_{sig} = 60 [dB]
If the gain error should be suppressed below
$\Delta G/G$ = 1% = 0.01 = -40 [dB],
Required GBP ω_u = 1[MHz]*1000 = 1[GHz]

NOTE: The gain error does not depends on ω_p.

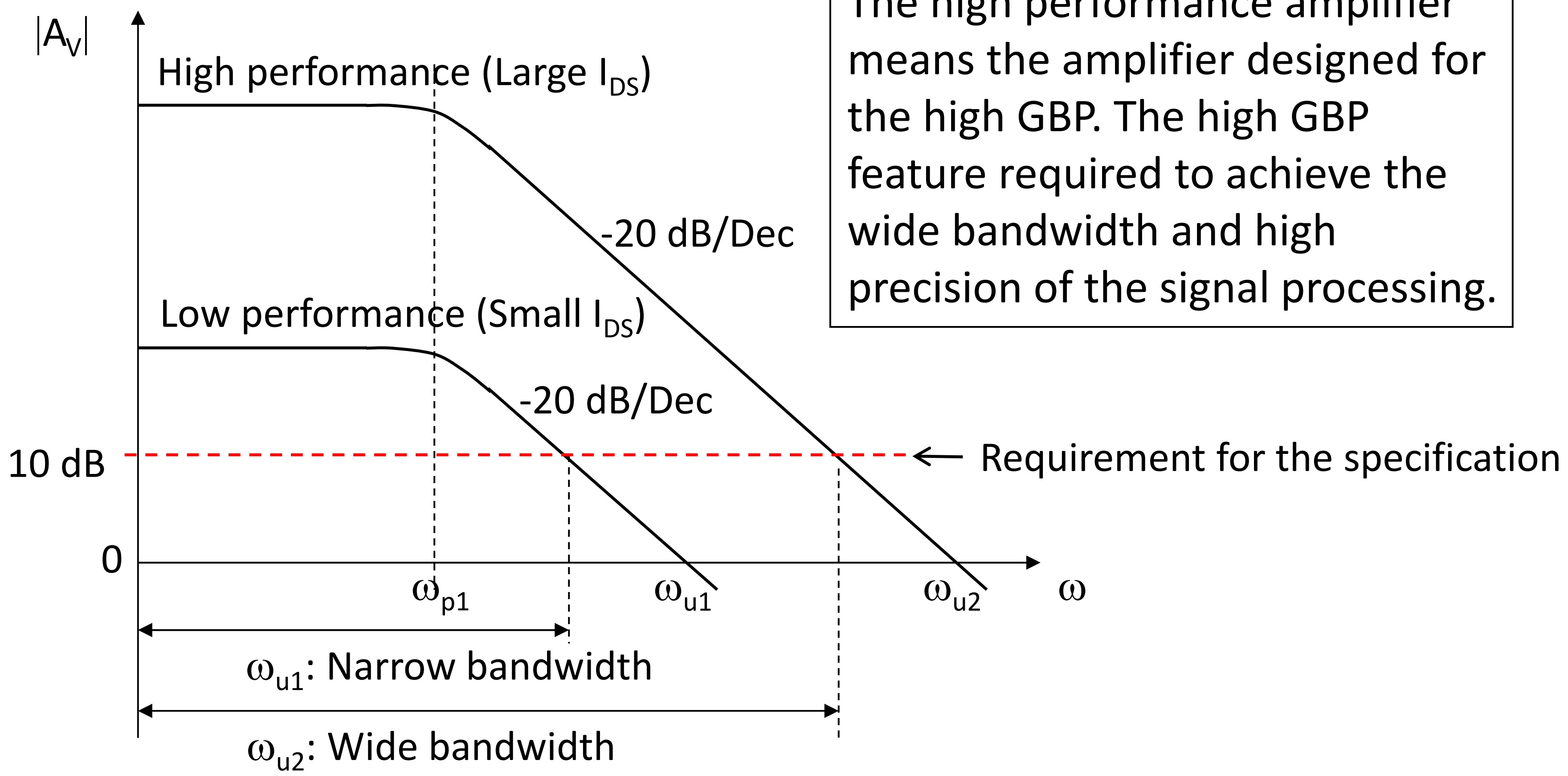
Frequency dependence of the gain error



The gain error is increased in high frequency, because the gain A is reduced.

The large GBP guarantees the low gain error of the circuit in the wide frequency range.

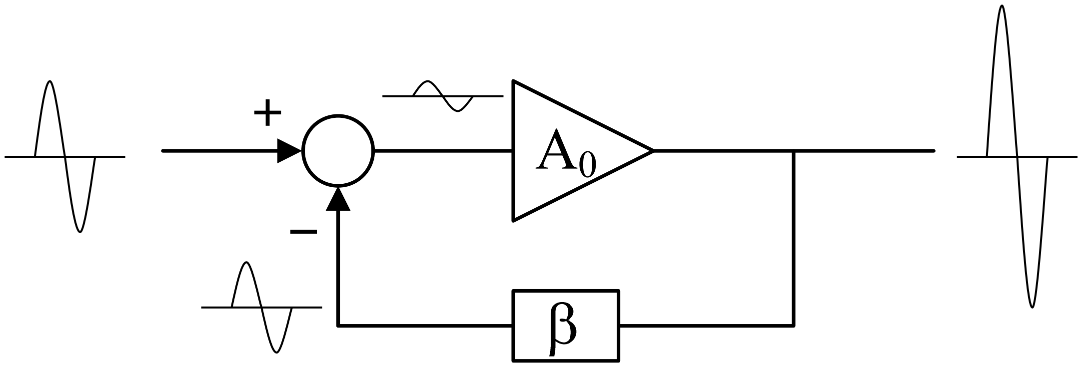
GBP as a figure of merit (FOM) of amplifiers



The high performance amplifier means the amplifier designed for the high GBP. The high GBP feature required to achieve the wide bandwidth and high precision of the signal processing.

4. Phase compensation

NFB (Negative Feedback)



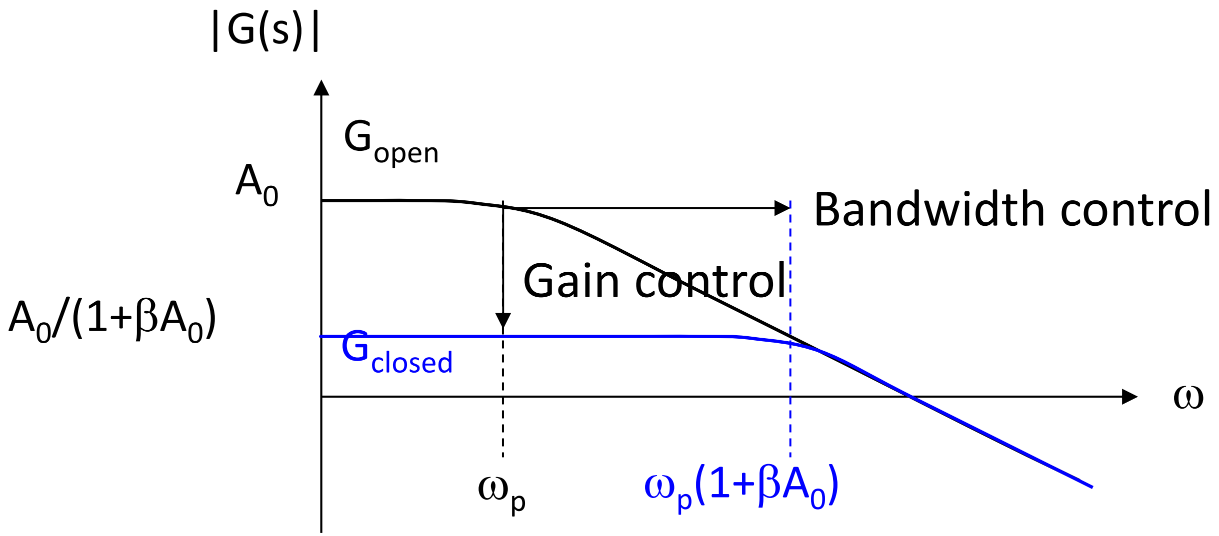
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\beta \cdot A_0} \underset{A_0 \to \infty}{\cong} \frac{1}{\beta}$$

Harold Black, US Patent 1921

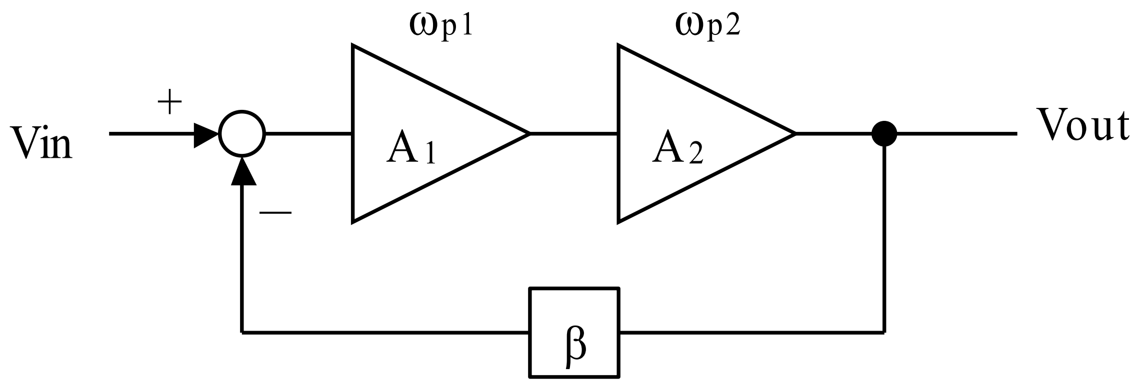
1. Precise control of transfer functions and stabilization of the gain
2. Suppression of the distortion
3. Extension of the frequency range
4. Suppression of the noise output to the output
5. Control of the input resistance and output resistance

Control of the gain and bandwidth

$$\begin{cases} G_{open}(s) = \dfrac{A_0}{1 + s/\omega_p} \\ G_{closed}(s) = \dfrac{A_0}{1 + \beta \cdot A_0} \dfrac{1}{1 + s/\omega_p(1 + \beta \cdot A_0)} \end{cases}$$



NFB applied to multi-stage amplifier



$$A_{total} = \frac{A_1 A_2}{1 + \beta \cdot A_1 A_2} \cong \frac{1}{\beta} \qquad (A_1 A_2 \doteqdot \infty)$$

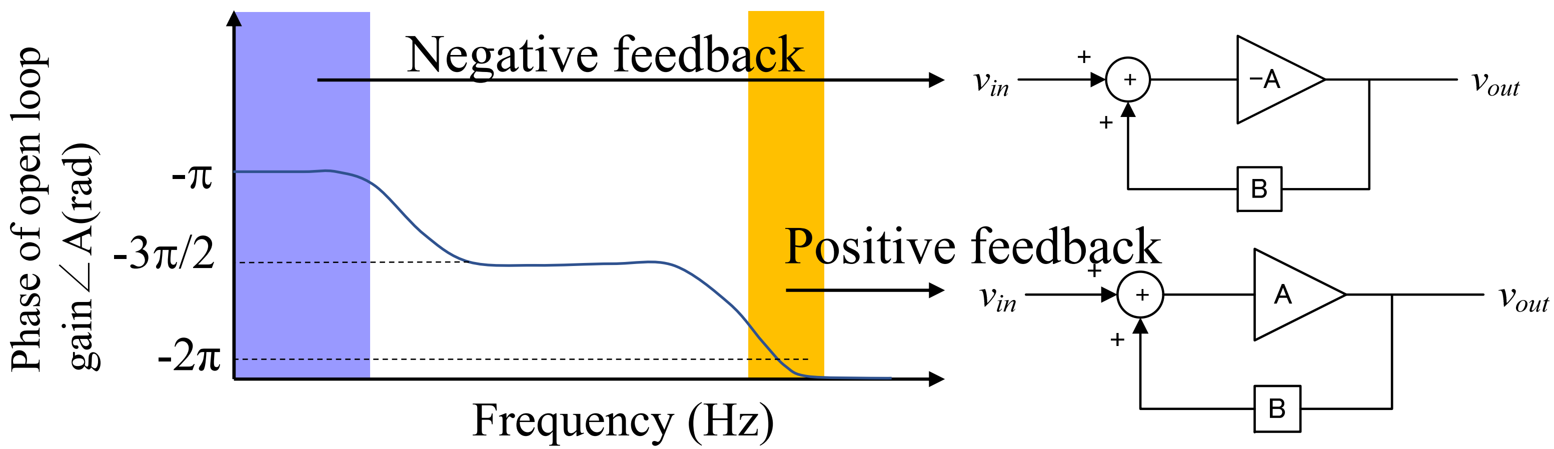
The effect of NFB is remarkable for the multi-stage amplifier, but ω_{p1} and ω_{p2} may be allocated in the neighbor frequency.

Stability of the NFB circuit

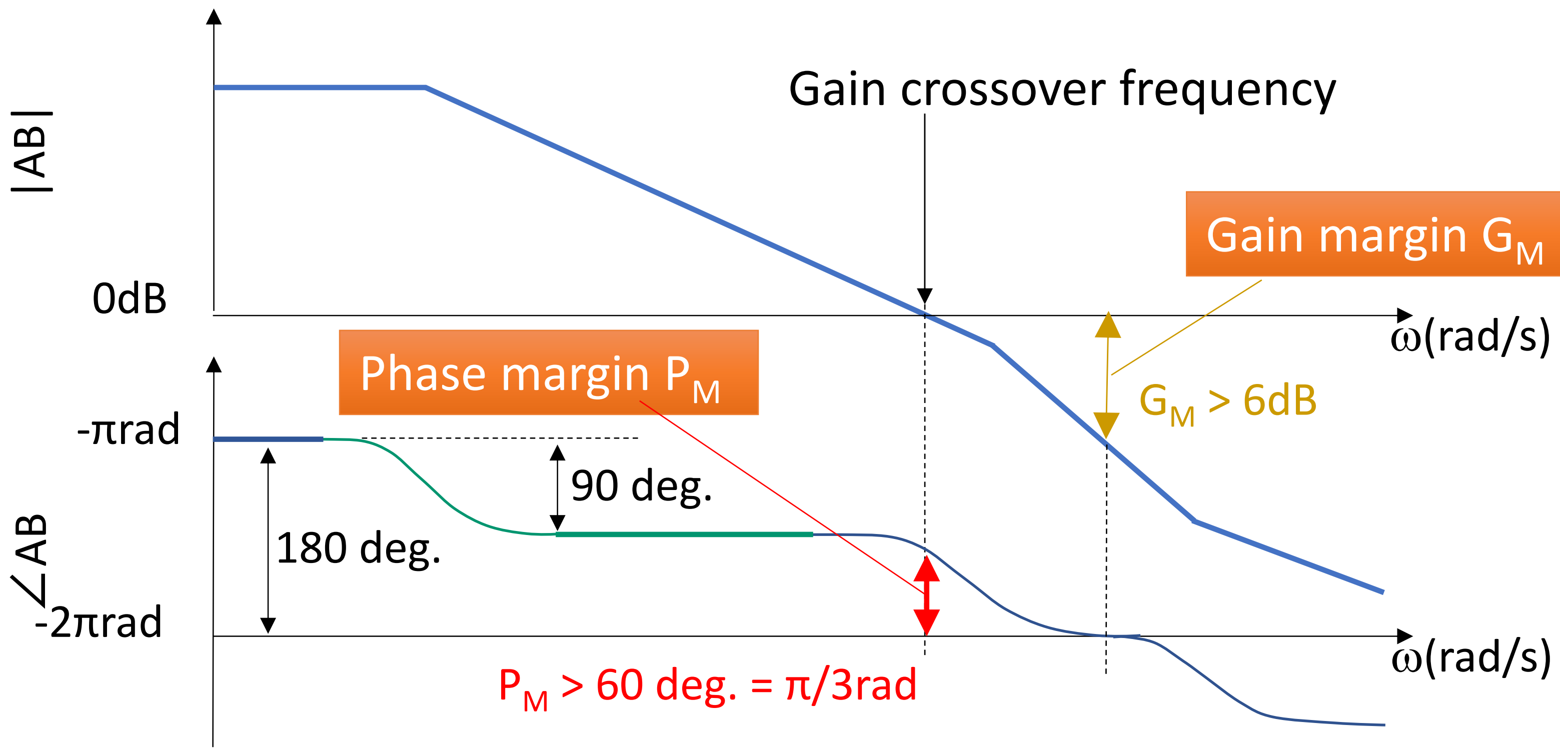
Second ω_p or ω_z may causes the positive feedback and there is a problem in the circuit stability.

- Long transient response
- Oscillation

The phase change of the loop gain should not exceed 180 degrees.

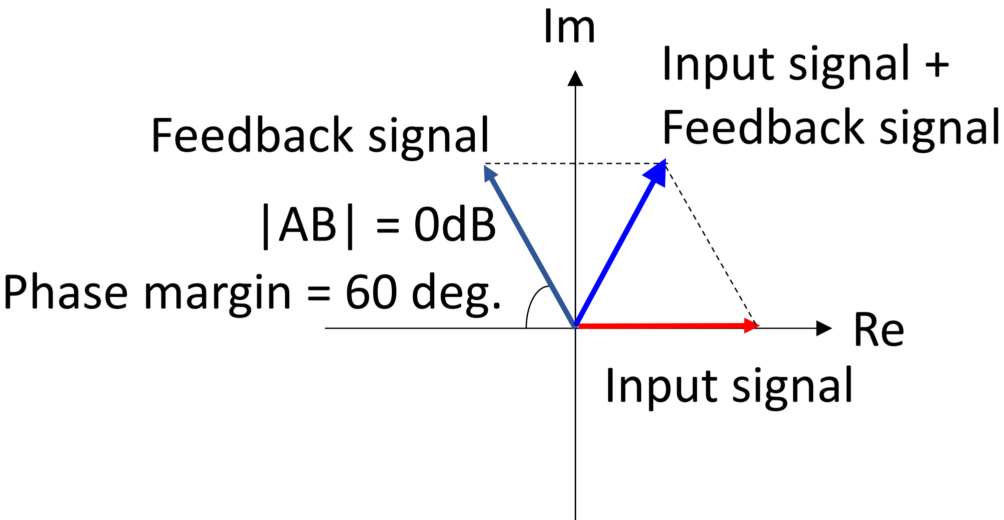


Stability condition



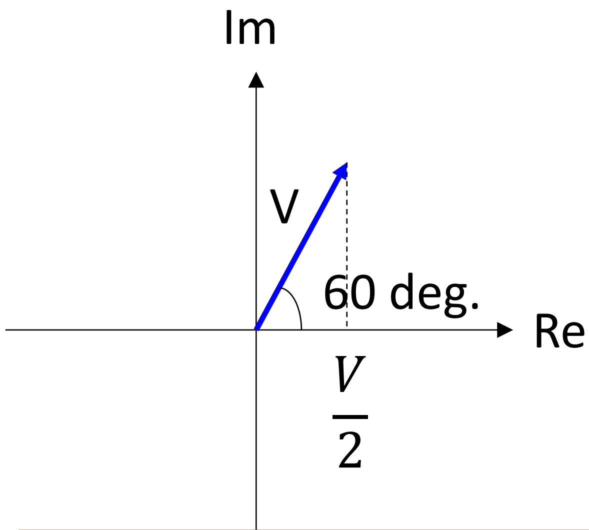
Gain margin and phase margin

Stability condition of P_M



When the P_M = 60 deg. and the loop gain = 0dB (100%), the resultant vector of the input signal and the feedback signal has the same magnitude as the magnitude of input signal.

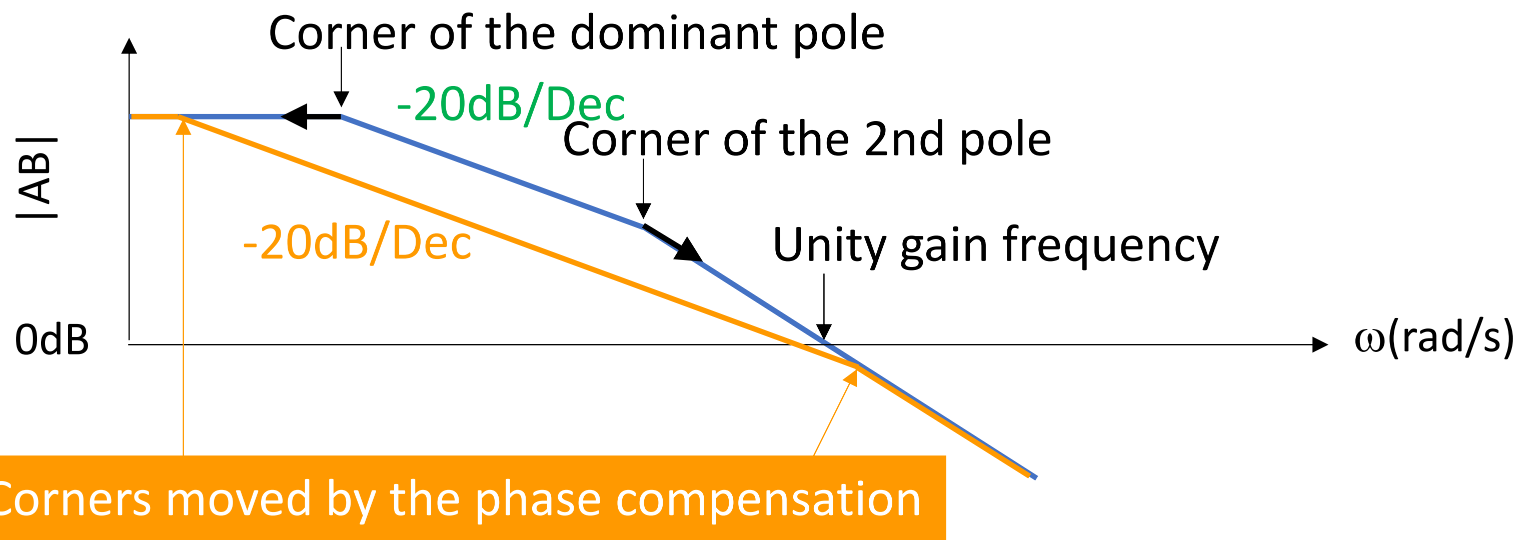
P_M and G_M



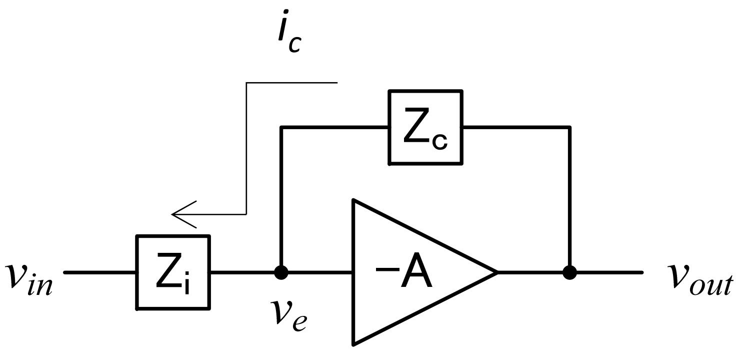
The 0.5 times change (-6dB) of the magnitude is equivalent to 60 deg. change of the phase.

Phase compensation

The 2-stage amplifier has 2 or more corners in the frequency response, that is, the phase rotation may exceed 180 degrees.
The phase compensation (位相補償) is used to increase the phase margin or the gain margin.



Example of phase compensation 1



The corner of the amplifier is moved by the feedback technique with Z_i and Z_c. The denominator of the transfer function is affected by the feedback loop.

$$A = \frac{A_0}{(1 + j\,\omega/\omega_{p1})\,(1 + j\,\omega/\omega_{p2})}$$

2 corners

$$\begin{cases} v_e - v_{in} = Z_i i_c \\ v_{out} - v_e = Z_c i_c \\ v_{out} = -A v_e \end{cases} \Rightarrow \quad i_c = \frac{1}{Z_c}\left(1 + \frac{1}{A}\right) v_{out} \cong \frac{1}{Z_c} v_{out} \quad (A \gg 1)$$

$$Gain = \frac{v_{out}}{v_{in}} = \frac{-A}{1 + \frac{Z_i}{Z_c} A} = \frac{-A_0}{(1 + j\,\omega/\omega_{p1})\,(1 + j\,\omega/\omega_{p2}) \underbrace{\boxed{+ \frac{Z_i}{Z_c} A_0}}_{\text{Added}}}$$

Example of phase compensation 2

$$Gain = \frac{-A_0}{(1 + j\,\omega/\omega_{p1})\,(1 + j\,\omega/\omega_{p2}) + \frac{Z_i}{Z_c} A_0}$$

$$= \frac{-A_0}{1 + j\omega\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) - \frac{\omega}{\omega_{p1}}\frac{\omega}{\omega_{p2}} + \frac{Z_i}{Z_c} A_0}$$

The corner of the 2nd pole ω_{p2} is multiplied by x.

$$= \frac{-A_0}{1 + j\omega\left(\frac{1}{\omega_{p1}/x} + \frac{1}{\omega_{p2} \cdot x}\right) - \frac{\omega}{\omega_{p1}}\frac{\omega}{\omega_{p2}} + j\omega\left(\frac{1-x}{\omega_{p1}} + \frac{1-\frac{1}{x}}{\omega_{p2}}\right) + \frac{Z_i}{Z_c} A_0}$$

$$j\omega\left(\frac{1-x}{\omega_{p1}} + \frac{1-\frac{1}{x}}{\omega_{p2}}\right) + \frac{Z_i}{Z_c} A_0 = 0$$

When $Z_i = R_i,\quad Z_c = \frac{1}{j\omega C_c},\qquad \frac{Z_i}{Z_c} A_0 = j\omega C_c R_i A_0 = -j\omega\left(\frac{1-x}{\omega_{p1}} + \frac{1-\frac{1}{x}}{\omega_{p2}}\right)$

Quiz

The amplifier with the following characteristics, ω_{p1} = 1MEGrad/s, ω_{p2} = 10MEGrad/s, A_0 = 40dB, R_i = 3.7MEGΩ.
Find the value of the capacitor Cc in order to move the corner to ω_{p2} = 100MEGrad/s.

$$\omega_{p2} = 10\text{MEG}rad/s \xrightarrow{x=10} = \omega_{p2} = 100\text{MEG}rad/s$$

$$j\omega C_c R_i A_0 = -j\omega\left(\frac{1-x}{\omega_{p1}} + \frac{1-\frac{1}{x}}{\omega_{p2}}\right)$$

$$C_c = -\frac{1}{R_i A_0}\left(\frac{1-x}{\omega_{p1}} + \frac{1-\frac{1}{x}}{\omega_{p2}}\right) = -\frac{1}{3.7\text{MEG}\cdot 100}\left(\frac{1-10}{1\text{MEG}} + \frac{1-\frac{1}{10}}{10\text{MEG}}\right)$$

$$= 24.1fF$$