11. Phase compensation

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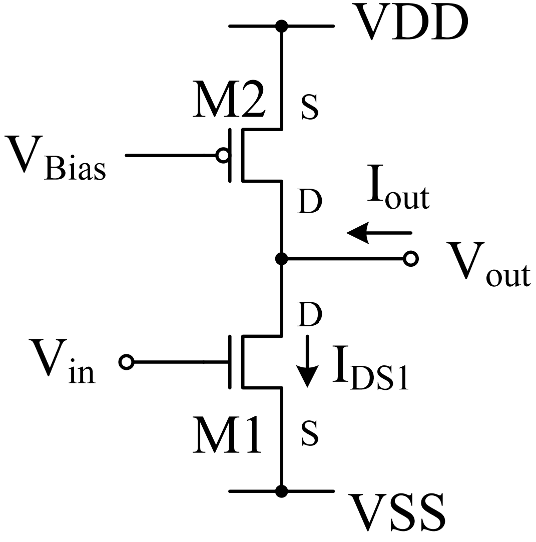
11.1 AC characteristics of CS amplifier

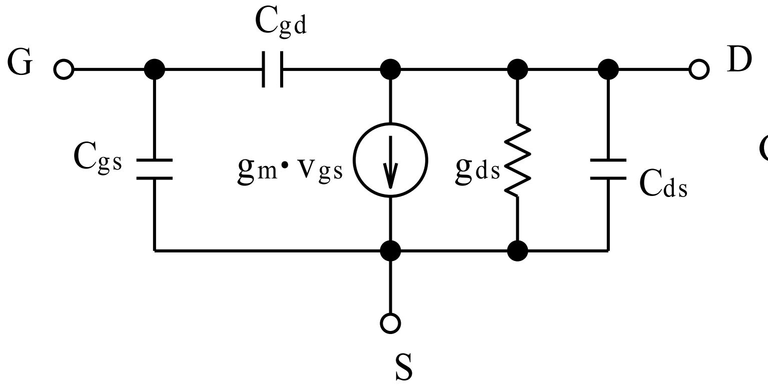
- A main factor to decide an AC characteristic
  - Output capacitance
    - Capacitive Load + Parasitic capacitance
  - Input capacitance
    - Parasitic capacitance
  - Input-output capacitance
    - Parasitic capacitance

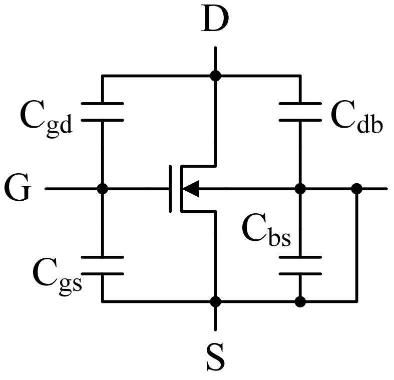
(AC characteristic: The small-signal frequency response)

Parasitic capacitance

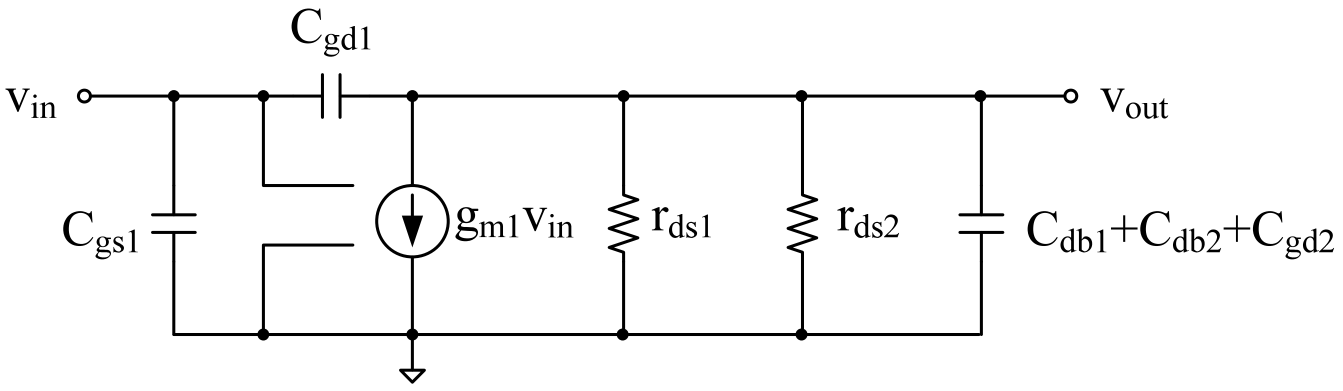


High frequency small-signal equivalent circuit of MOSFET

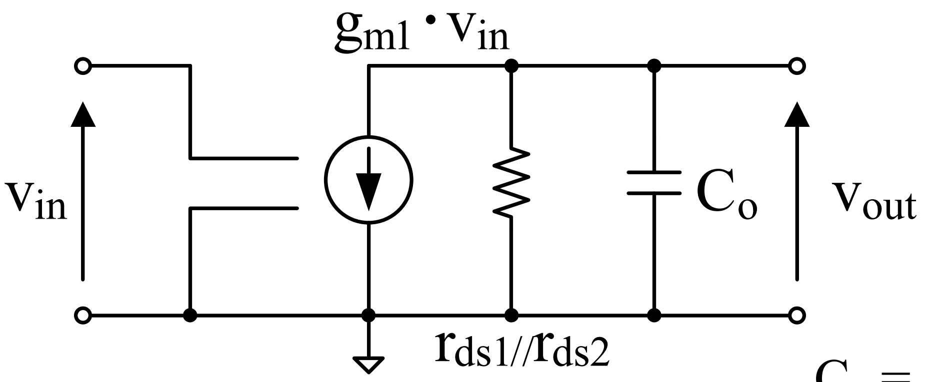




High frequency small-signal equivalent circuit of CS amplifier



Influence of the output capacitance



$C_o = C_{db1} + C_{db2} + C_{gd2} + C_L$

$$v_{out} = \frac{1}{\dfrac{1}{r_{ds1} // r_{ds2}} + j\omega \cdot C_O}(-g_m v_{in})$$

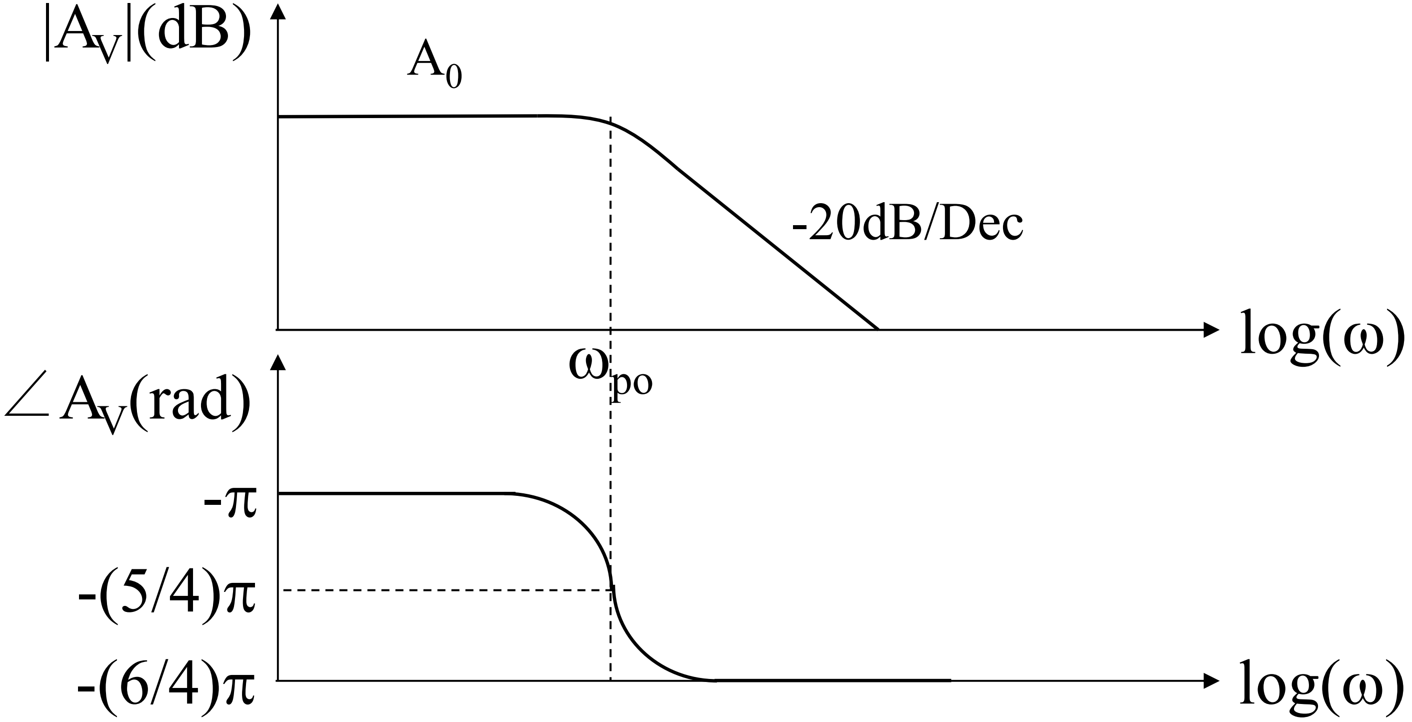
$$A(\omega) \equiv \frac{v_{out}}{v_{in}} = \frac{-g_m(r_{ds1} // r_{ds2})}{1 + j\omega \cdot C_O(r_{ds1} // r_{ds2})}$$

$$= \frac{-A_0(\omega = 0)}{1 + j\omega \cdot C_O(r_{ds1} // r_{ds2})}$$

Bode diagram of the CS amplifier

$$|A(\omega)| = \frac{A_0}{\sqrt{1+\omega^2/\omega_{po}^2}} \qquad \omega_{po} = \frac{1}{C_o(r_{ds1}//r_{ds2})}$$

(pole frequency of output)



Bias dependence of a pole frequency

$$\begin{cases} g_{m1} = \sqrt{2\beta_1 I_{DS1}} \\ r_{ds1} // r_{ds2} = \dfrac{1}{g_{ds1} + g_{ds2}} = \dfrac{1}{(\lambda_1 + \lambda_2) \cdot I_{DS1}} \end{cases}$$

$$A_0 = g_{m1}(r_{ds1} // r_{ds2}) = \frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2} \frac{1}{\sqrt{I_{DS1}}}$$

(DC gain)

$$\omega_{po} = \frac{1}{C_o(r_{ds1} // r_{ds2})} = \frac{(\lambda_1 + \lambda_2) \cdot I_{DS1}}{C_o}$$

(pole frequency)

$$\omega_{po} \cdot A_0^2 = \frac{1}{C_o} \frac{2\beta}{(\lambda_1 + \lambda_2)}$$

The product of the ω_{po} and A_0^2 is independent on the bias current.

Unity gain frequency ω_u (= BGP)

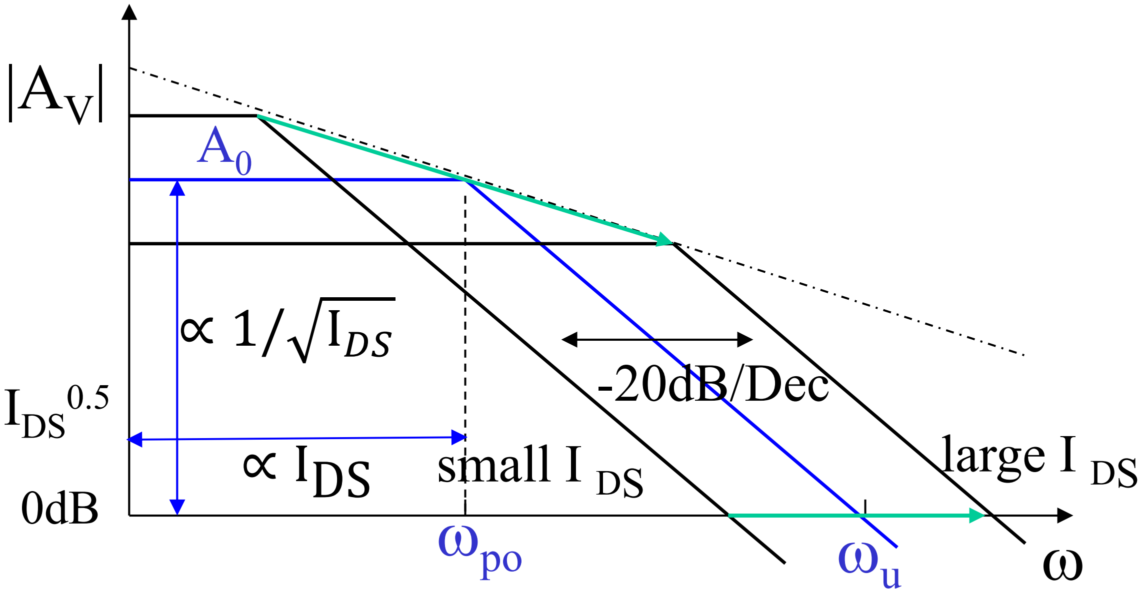
$$\frac{A_0}{\sqrt{1+\omega_u^2/\omega_{po}^2}}=1$$

$$\omega_u=\sqrt{(A_0^2-1)}\omega_{po}$$

$$\cong A_0\omega_{po}=\frac{g_m}{C_o}$$

$\propto I_{DS}^{0.5}$

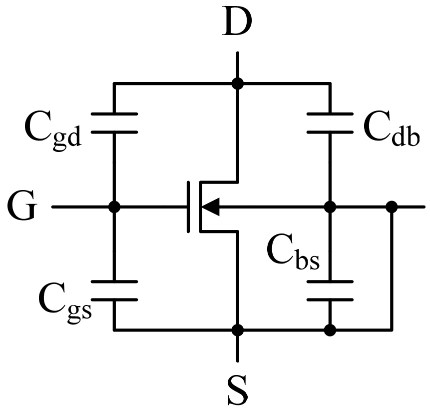
$$=\frac{\beta(V_{GS}-V_T)}{C_o}$$



I_D ↑: A_0↓, ω_{po}↑, ω_u↑

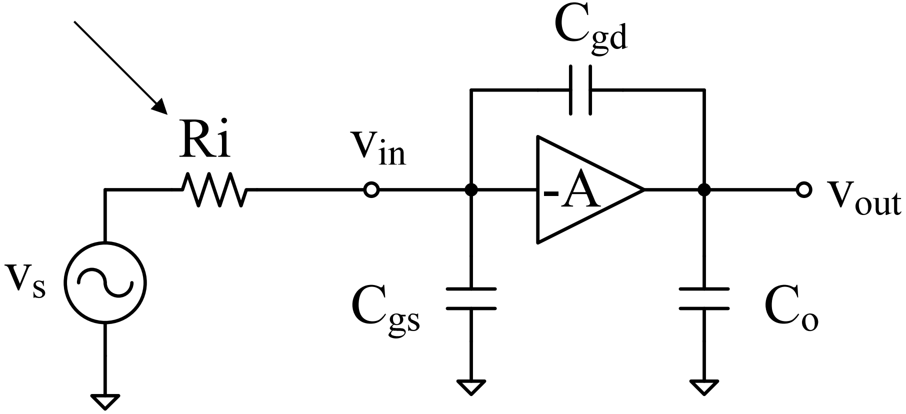
NOTE: ω_u ≒ GBP (Gain Bandwidth Product)

Influence of the input capacitance



Output resistance of preceding stage

Voltage gain -A



Miller effect

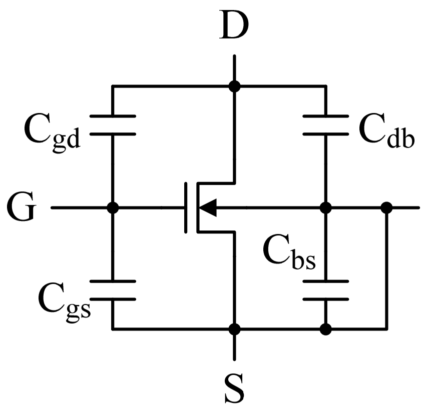
$$C_i = C_{gs1} + C_{gd1}(1+A) \cong AC_{gd1}$$

$$C_O = C_{db1} + C_{db2}\ \ C_{gd2} + C_L$$

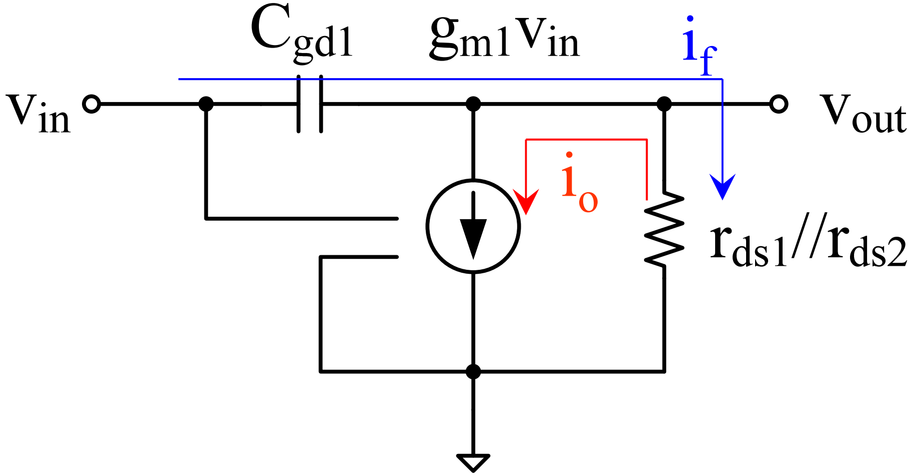
$$\omega_{pi} \cong \frac{1}{AR_iC_{gd1}}$$ (pole frequency of input)

NOTE: The pole frequency is low when a voltage gain is large.

Influence of the input-output capacitance (1)



Current path through C_{gd}



$$\begin{cases} v_{out} = (r_{ds1} // r_{ds2})(i_f - i_o) \\ i_f = j\omega \cdot C_{gd}(v_{in} - v_{out}) \end{cases}$$

Influence of the input-output capacitance (1)

by competitive current i_o and i_f

Normally $(r_{ds1}//r_{ds2}) < 1/g_{m1}$,
$\omega_z < \omega_{pgd}$

$$\frac{v_{out}}{v_{in}} = \frac{-(r_{ds1}//r_{ds2})g_{m1}(1 - j\omega\cdot\frac{C_{gd}}{g_{m1}})}{1 + j\omega\cdot C_{gd}(r_{ds1}//r_{ds2})} \equiv \frac{-(r_{ds1}//r_{ds2})g_{m1}(1 - j\omega/\omega_z)}{1 + j\omega/\omega_{pgd}}$$

i_f : Forward transmission signal from G to D
i_o : Normally amplified signal

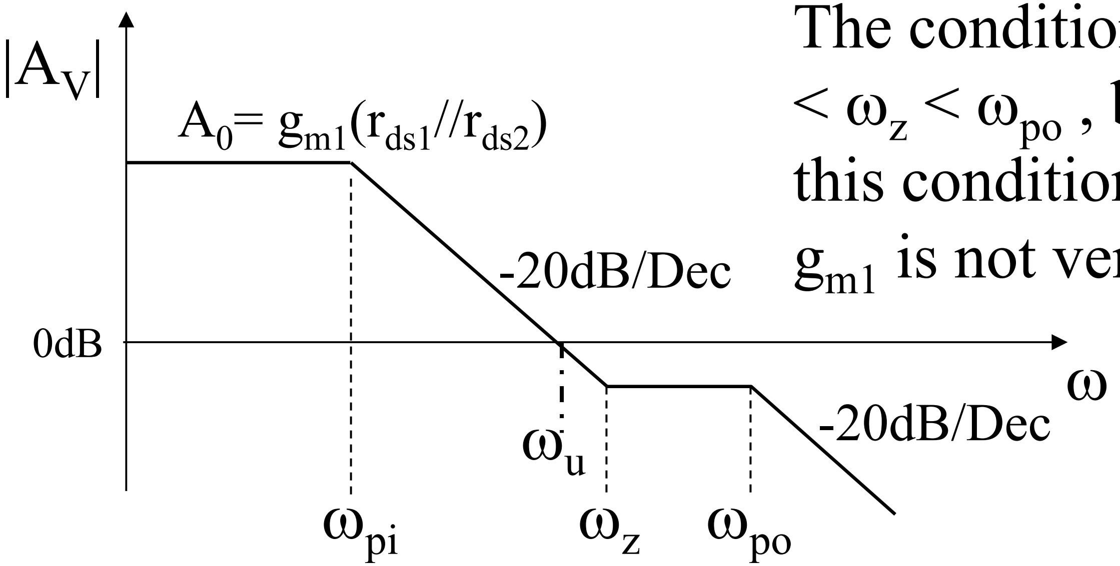
The balance of i_f and i_o generate the zero.

Summary of AC characteristics of the CS amplifier

$$A_V = \frac{A_0(1 - j\omega/\omega_z)}{(1 + j\omega/\omega_{pi})(1 + j\omega/\omega_{po})}$$

2 -pole and 1-zero transfer function

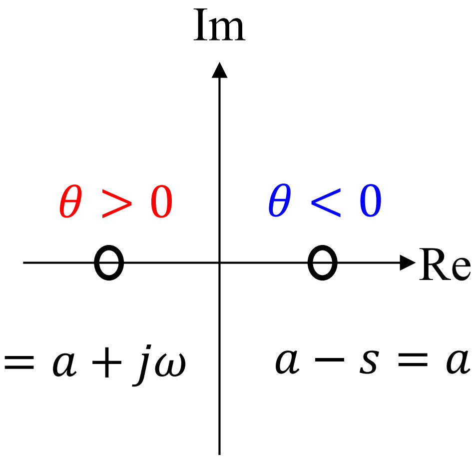
(The ω_{pgd} is placed in the very high frequency, thus it is usually negligible.)



The condition may not keep $\omega_{pi} < \omega_z < \omega_{po}$, but it is often that this condition is observed when g_{m1} is not very large.

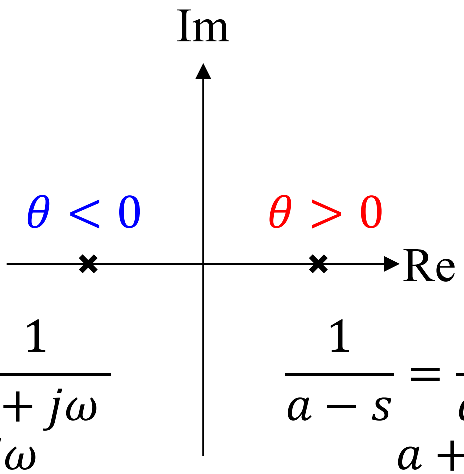
NOTE: Pole in the left half plane and zero in the right half plane turns phase -90 degrees.

Phase characteristic



$a + s = a + j\omega$

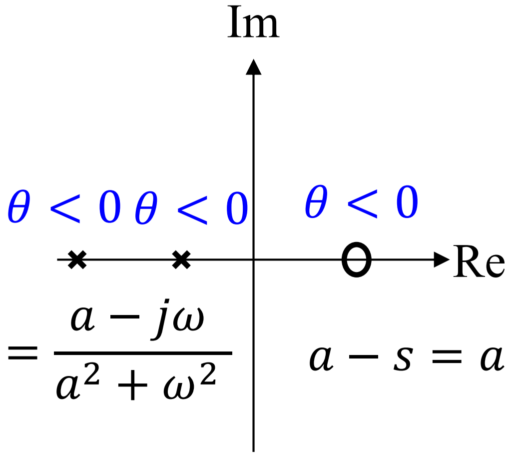
$a - s = a - j\omega$



$$\frac{1}{a+s} = \frac{1}{a+j\omega} = \frac{a-j\omega}{(a+j\omega)(a-j\omega)}$$

$$\frac{1}{a-s} = \frac{1}{a-j\omega} = \frac{a+j\omega}{(a-j\omega)(a+j\omega)}$$

CS amplifier



$$\frac{1}{a+s} = \frac{a-j\omega}{a^2+\omega^2}$$

$a - s = a - j\omega$

All pole and zero retard the negative phase of transfer function.

11.2 AC characteristics of cascode amplifier

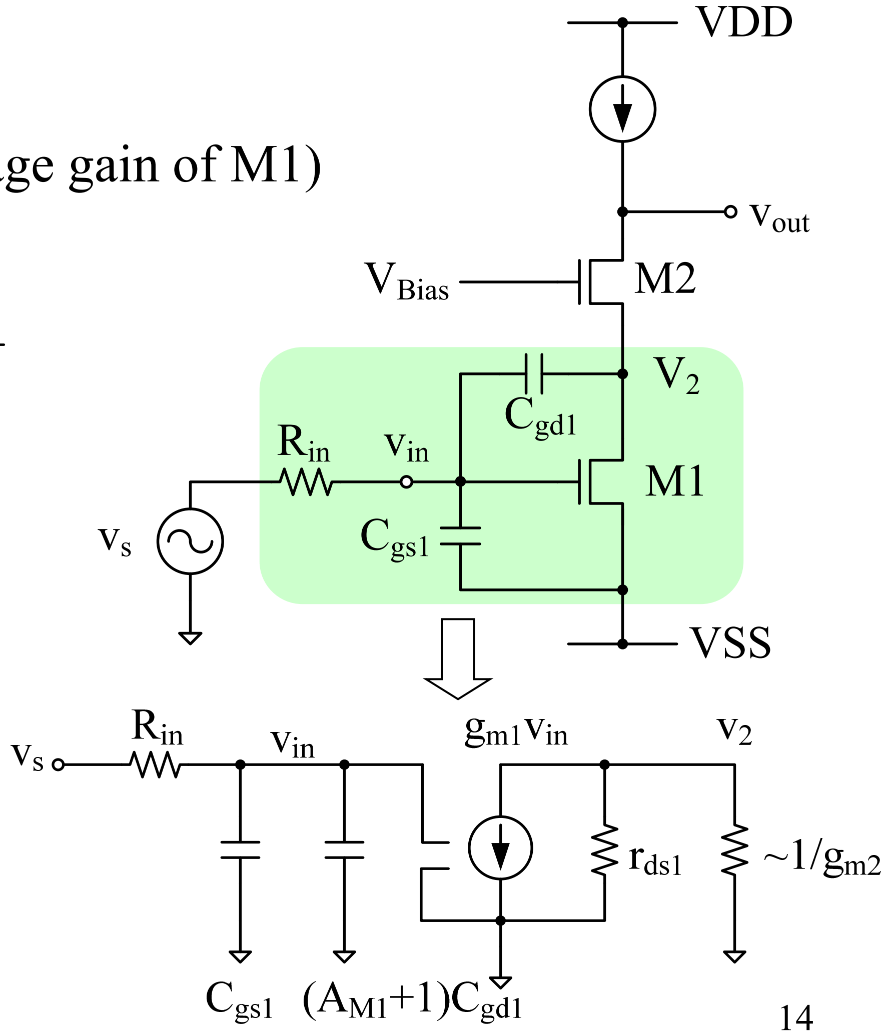
Influence of the input capacitance

$$\begin{cases} A_{M1} = \dfrac{g_{m1} r_{ds1}}{1 + g_{m2} r_{ds1}} \approx \dfrac{g_{m1}}{g_{m2}} & \text{(Voltage gain of M1)} \\ \omega_{pi} = \dfrac{1}{R_{in}\{C_{gs1} + (1 + \dfrac{g_{m1}}{g_{m2}})C_{gd1}\}} \end{cases}$$

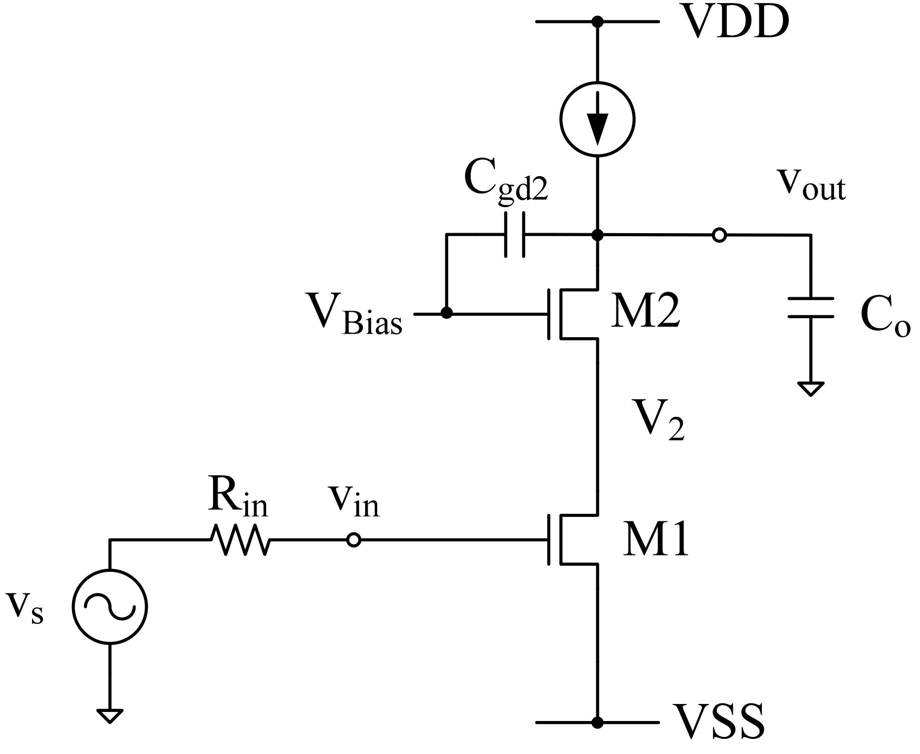
The voltage gain of M1 is almost unity, because the input resistance of M2 is 1/gm2.

⇩

Miller effect is negligible and ω_{pi} is very high.



Influence of the output capacitance



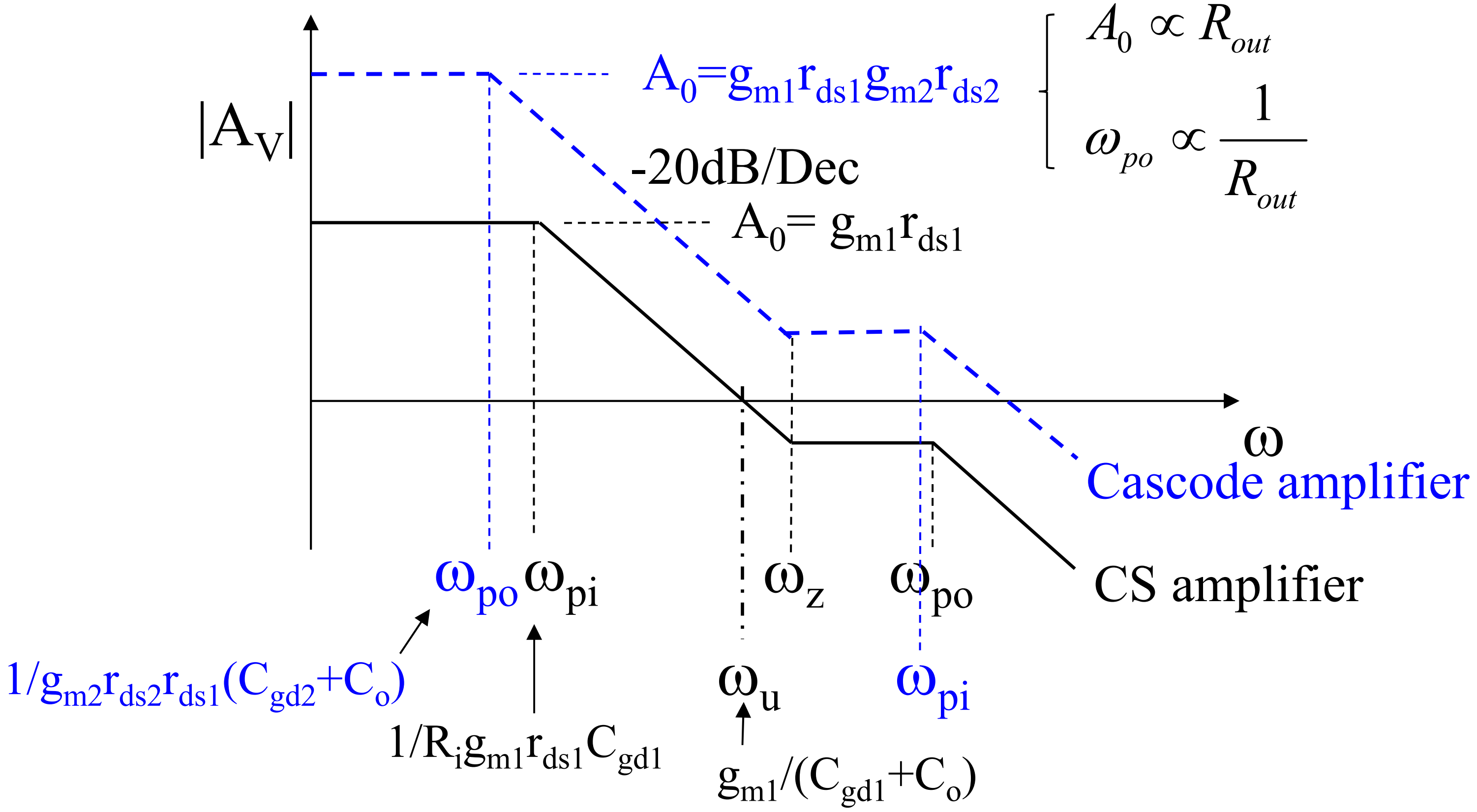
Output resistance of M2:

$$R_{out} = g_{m2} r_{ds2} r_{ds1}$$

$$\therefore \omega_{po} = \frac{1}{R_{out}(C_{gd2} + C_o)}$$

Normally $\omega_{po} < \omega_{pi}$, because the output resistance of cascode amplifier R_{out} is very large.

Comparison between Cascode amplifier and CS amplifier



$$\begin{cases} A_0 \propto R_{out} \\ \omega_{po} \propto \dfrac{1}{R_{out}} \end{cases}$$

11.3 AC performance of amplifiers

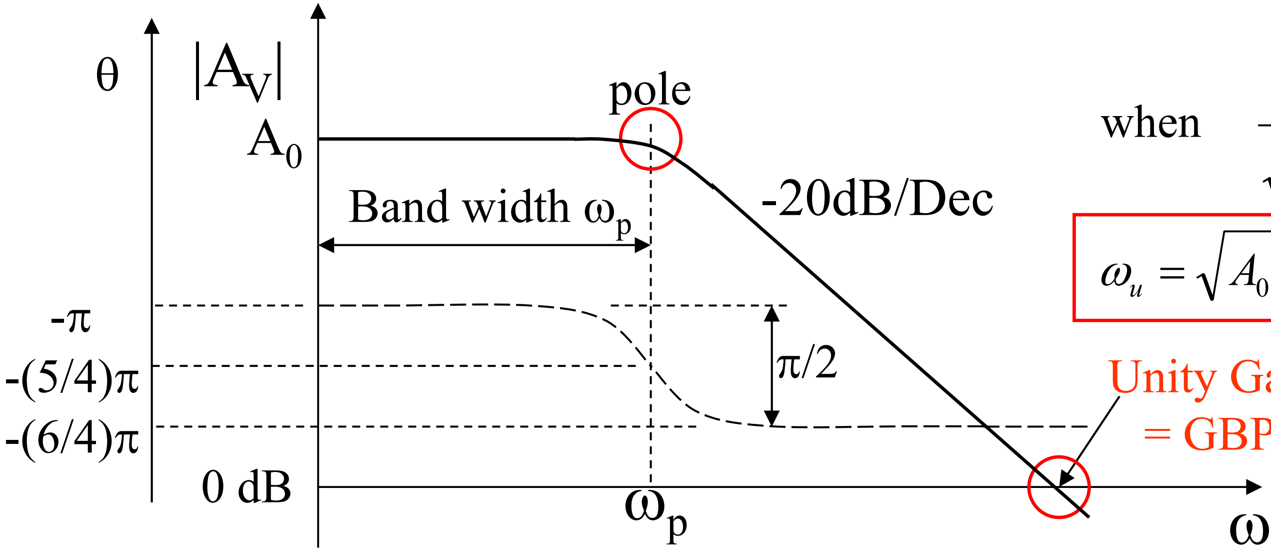
Simplified 1-pole model of AC characteristic of amplifiers

$$A_V(\omega) = \frac{A_0}{1+j\omega/\omega_p} = \frac{A_0\cdot\omega_p}{j\omega+\omega_p} \cong \frac{\omega_u}{j\omega+\omega_p}$$

$$|A_V(\omega)| = \left|\frac{v_{out}}{v_{in}}\right| = \frac{A_0}{\sqrt{1+\omega^2/\omega_p^2}}$$

ω_p: the pole frequency (or cut-off frequency)

ω_u: the unity gain frequency



when $$\frac{A_0}{\sqrt{1+\omega_u^2/\omega_p^2}} = 1$$

Important

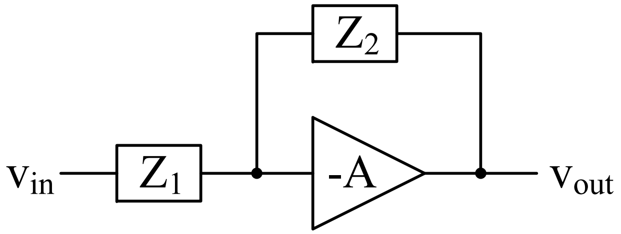
$$\boxed{\omega_u = \sqrt{A_0^2 - 1}\cdot\omega_{p1} \cong A_0\cdot\omega_p \equiv GBP}$$

Unity Gain Frequency ω_u
= GBP (Gain-bandwidth product)

Gain error of circuits and GBP

Gain of NFB amplifier

$$G = \frac{v_{out}}{v_{in}} = \frac{-\dfrac{Z_2}{Z_1}}{1+\dfrac{1+\dfrac{Z_2}{Z_1}}{A}} \xrightarrow{A\to\infty} -\frac{Z_2}{Z_1}$$



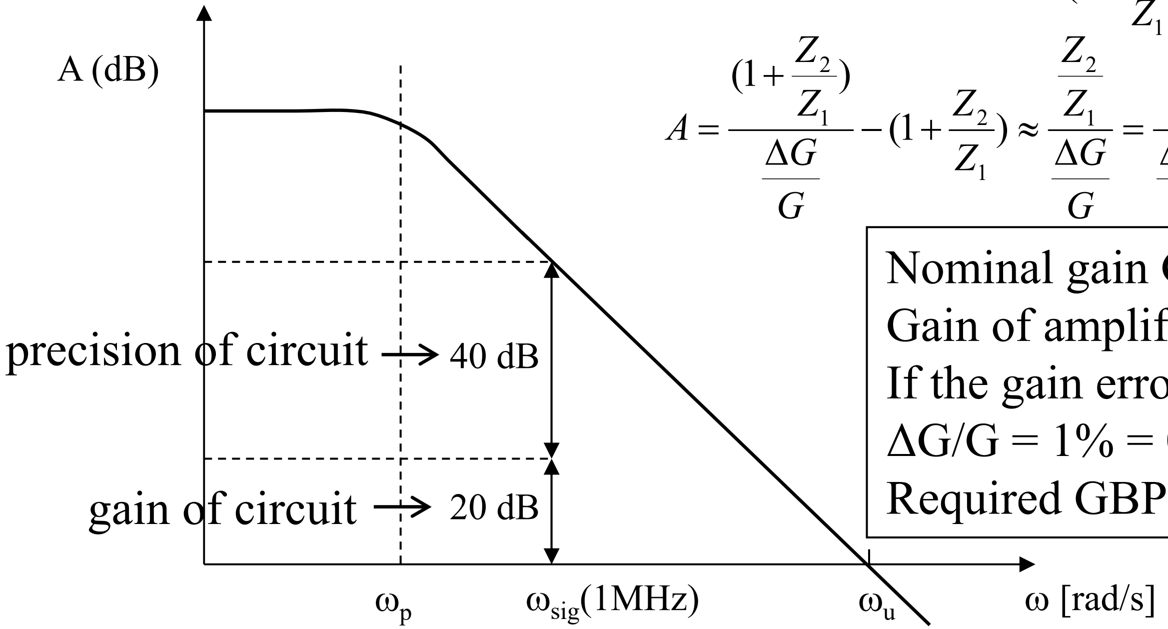
NFB amplifier

Gain error of NFB amplifier

$$\frac{\Delta G}{G} = \frac{G(\infty)-G(A)}{G(\infty)} = \frac{1+\dfrac{Z_2}{Z_1}}{A+(1+\dfrac{Z_2}{Z_1})}$$

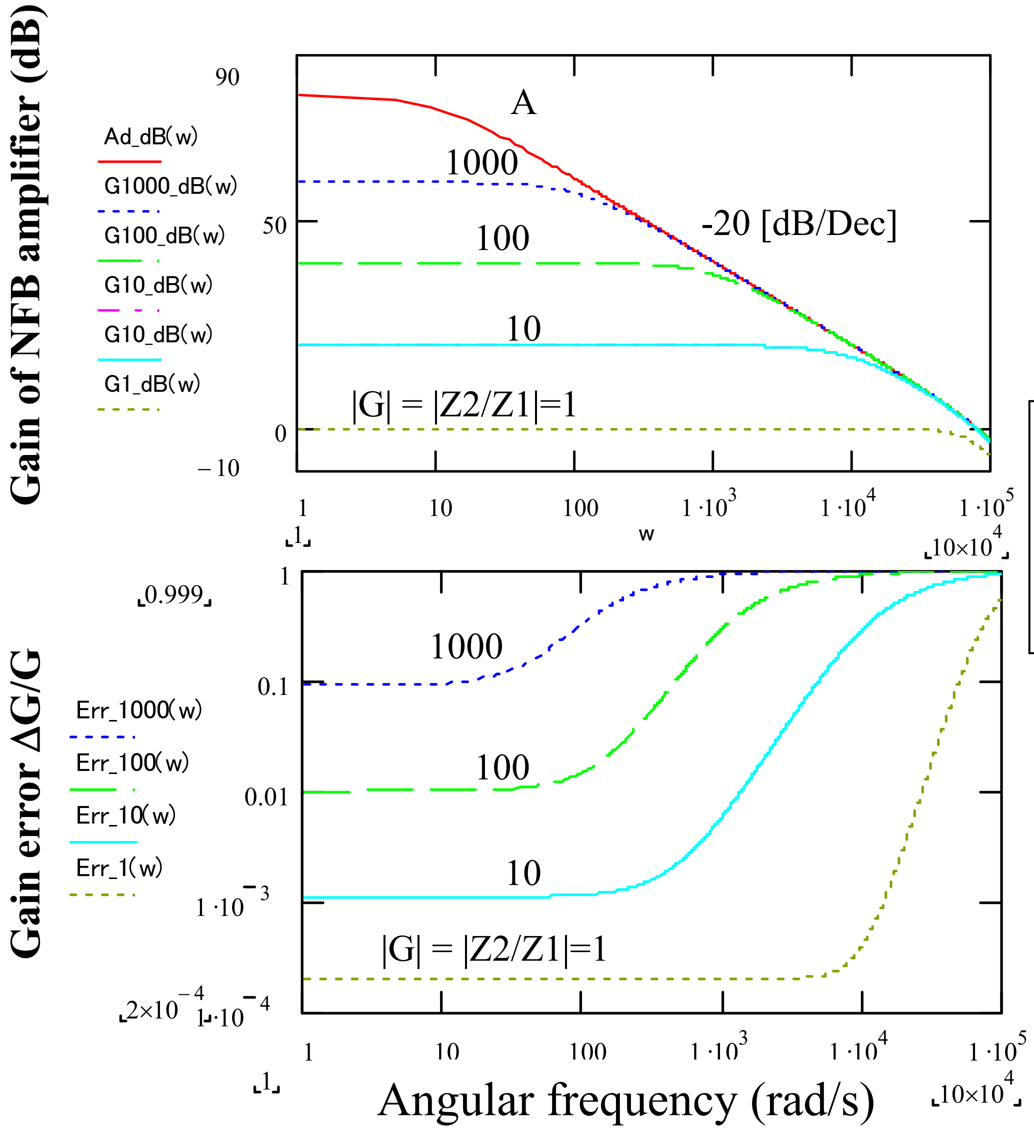
$$A = \frac{(1+\dfrac{Z_2}{Z_1})}{\dfrac{\Delta G}{G}} - (1+\frac{Z_2}{Z_1}) \approx \frac{\dfrac{Z_2}{Z_1}}{\dfrac{\Delta G}{G}} = \frac{G}{\dfrac{\Delta G}{G}} = G[dB] - \frac{\Delta G}{G}[dB]$$

Nominal gain G = 10.0 = 20 [dB] (Ideal value)
Gain of amplifier A at ω_{sig} = 60 [dB]
If the gain error should be suppressed below
ΔG/G = 1% = 0.01 = -40 [dB],
Required GBP ω_u = 1[MHz]*1000 = 1[GHz]



NOTE: The gain error does not depends on ω_p.

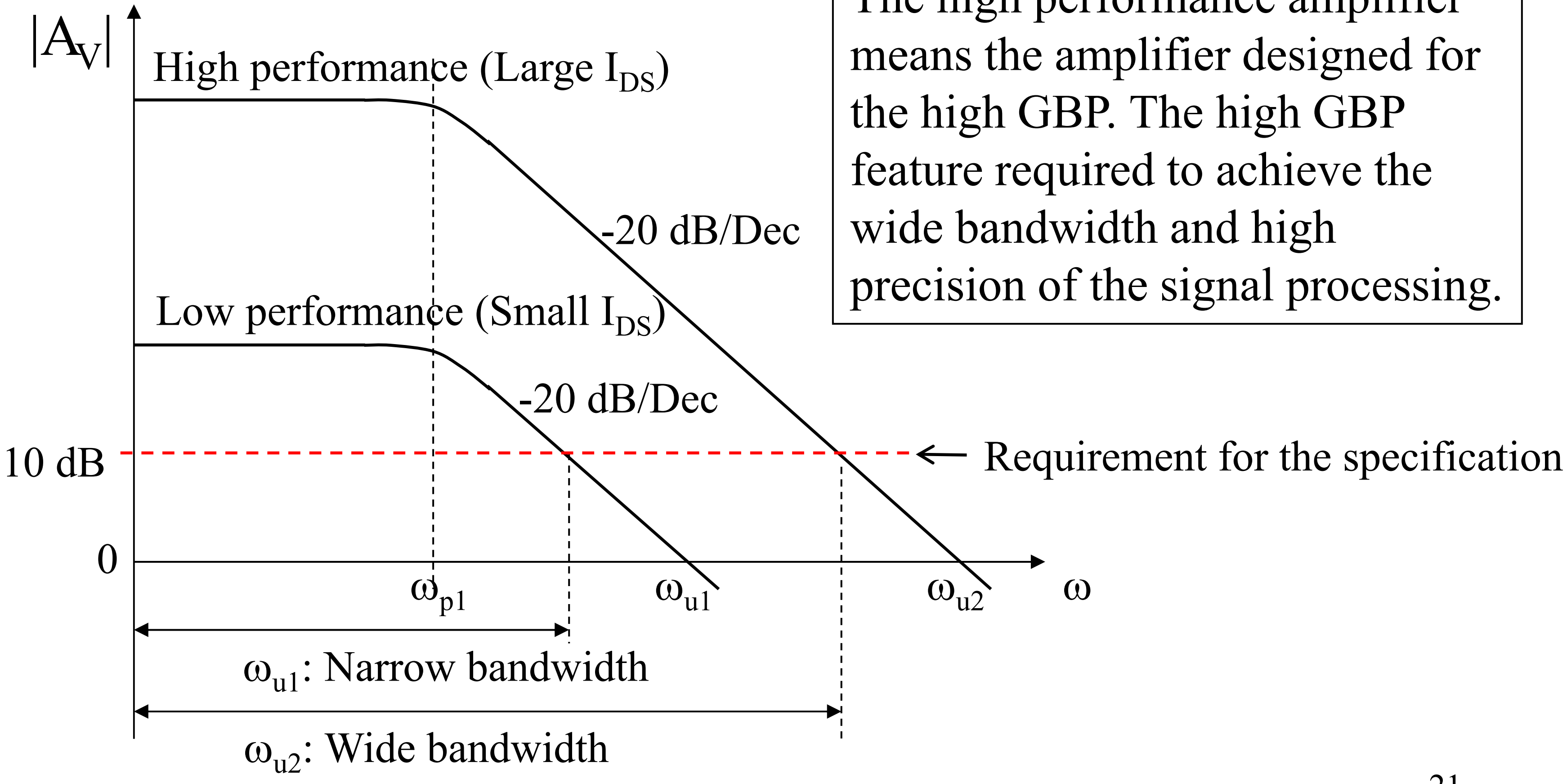
Frequency dependence of the gain error



The gain error is increased in high frequency, because the gain A is reduced.

The large GBP guarantees the low gain error of the circuit in the wide frequency range.

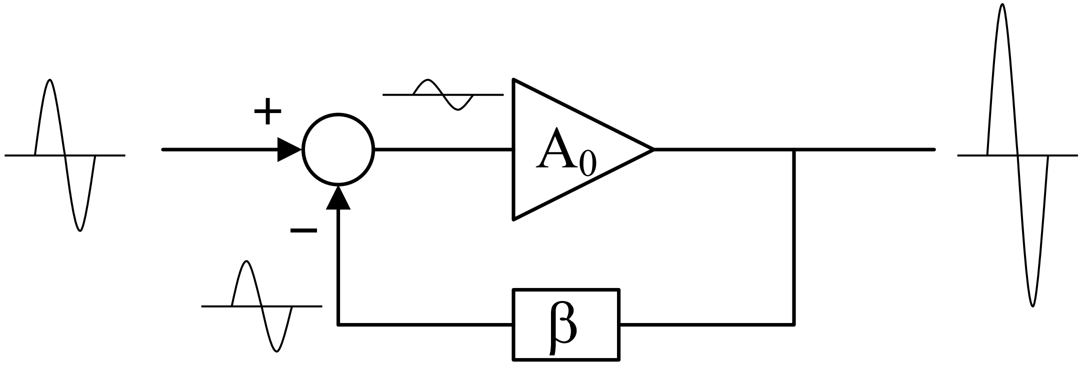
GBP as a figure of merit (FOM) of amplifiers



The high performance amplifier means the amplifier designed for the high GBP. The high GBP feature required to achieve the wide bandwidth and high precision of the signal processing.

11.4 Phase compensation

NFB (Negative Feedback)



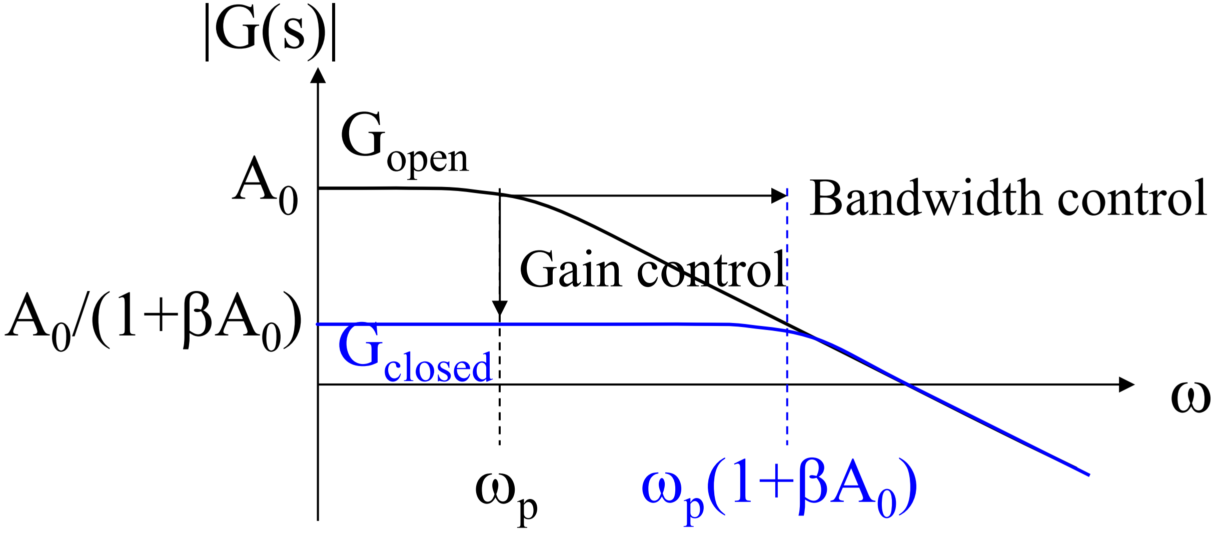
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\beta\cdot A_0} \overset{A_0\to\infty}{\cong} \frac{1}{\beta}$$

Harold Black, US Patent 1921

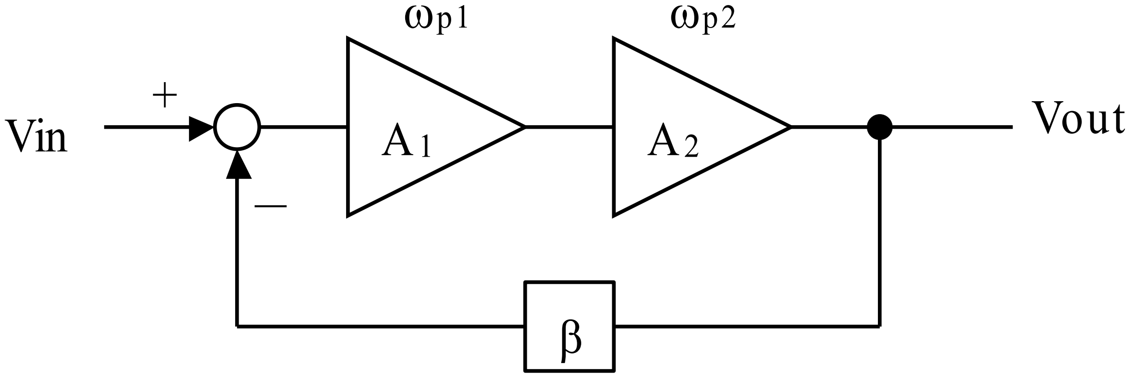
1. Precise control of transfer functions and stabilization of the gain
2. Suppression of the distortion
3. Extension of the frequency range
4. Suppression of the noise output to the output
5. Control of the input resistance and output resistance

Control of the gain and bandwidth

$$\begin{cases} G_{open}(s) = \dfrac{A_0}{1 + s/\omega_p} \\ G_{closed}(s) = \dfrac{A_0}{1 + \beta \cdot A_0} \dfrac{1}{1 + s/\omega_p(1 + \beta \cdot A_0)} \end{cases}$$



NFB applied to multi-stage amplifier



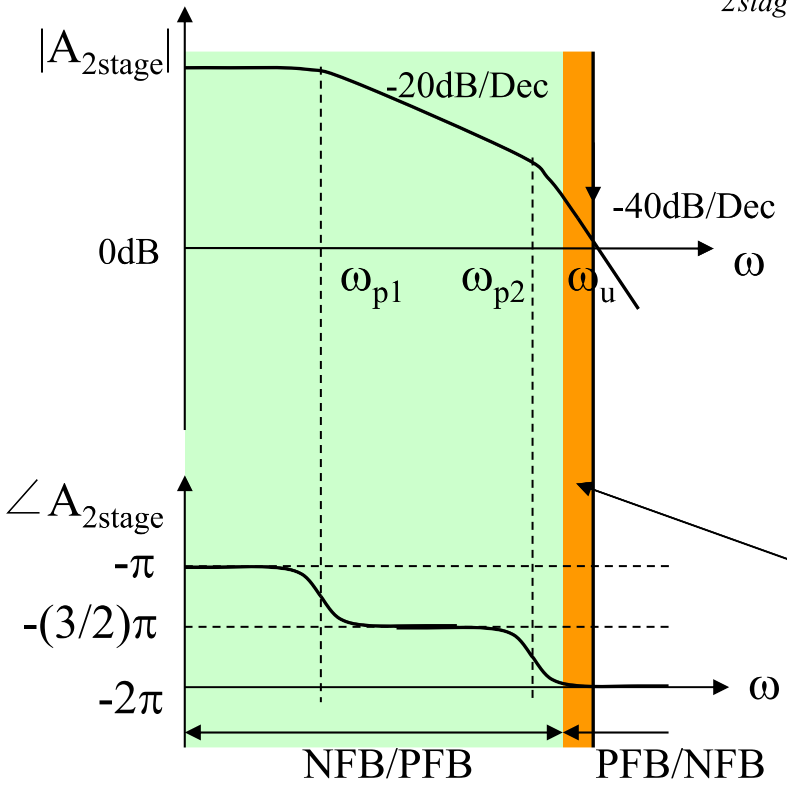
$$A_{total} = \frac{A_1 A_2}{1 + \beta \cdot A_1 A_2} \cong \frac{1}{\beta} \qquad (A_1 A_2 \doteqdot \infty)$$

The effect of NFB is remarkable for the multi-stage amplifier, but ω_{p1} and ω_{p2} may be allocated in the neighbor frequency.

Stability of the NFB circuit

Second ω_p or ω_z may causes the positive feedback and there is a problem in the circuit stability.

$$A_{2stage} = A_1(\omega)A_2(\omega) = \frac{A_0}{(1 + j\omega/\omega_{p1})(1 + j\omega/\omega_{p2})}$$



Oscillation condition of the loop gain

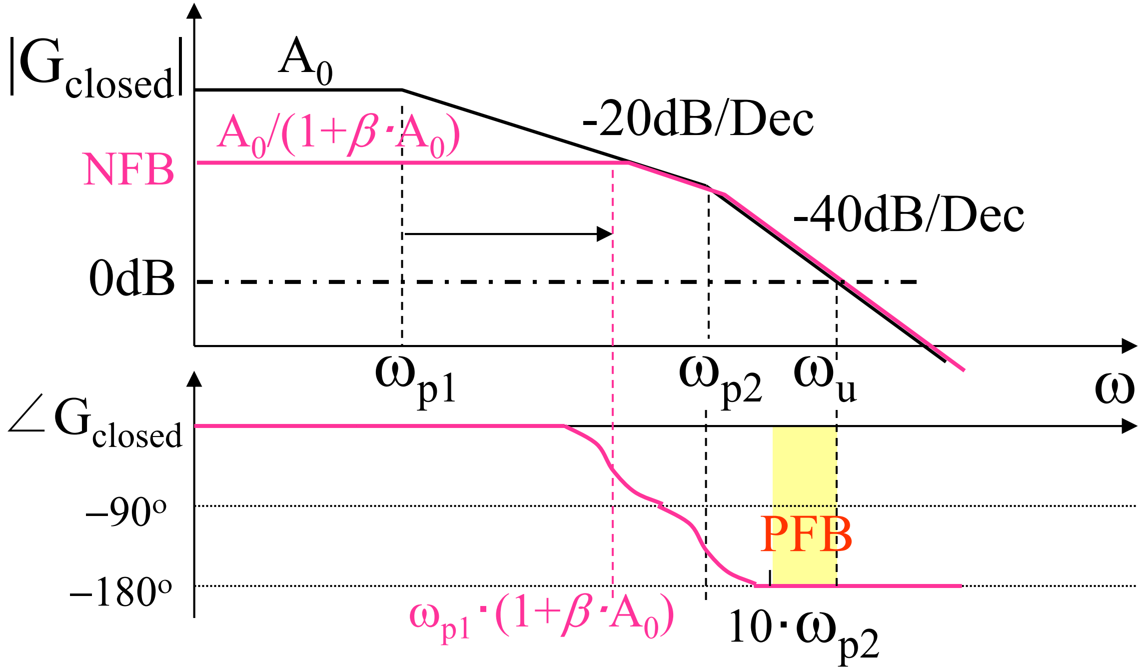
$$\begin{cases} -\beta \cdot A = \dfrac{-\beta \cdot A_0}{(1 + j\omega/\omega_{p1})(1 + j\omega/\omega_{p2})} \\ |-\beta \cdot A| \geq 0(dB) \\ \angle -\beta \cdot A = \pm 2\pi(rad) \end{cases}$$

In this frequency region, the NFB circuit works as a positive feedback (PFB) system.

Closed loop AC characteristic

$$A_{closed}(s) = \frac{A_{open}(s)}{1+\beta \cdot A_{open}(s)} = \frac{A_0(const.)}{(1+s/\omega_{p1})(1+s/\omega_{p2})+\beta \cdot A_0(const.)}$$

$$\cong \frac{A_0/(1+\beta \cdot A_0)}{1+s/\omega_{p1}(1+\beta \cdot A_0)}$$,where $s/\omega_{p1} >> s/\omega_{p2}$



Displacement of the pole frequency

$$A_{closed}(s) = \frac{A_0}{(1+s/P\omega_{p1})(1+s/Q\omega_{p2})} \longleftarrow$$

move the ω_{p1} and ω_{p2}

P and Q are set by the addition of feedback element β(s).

$$A_{closed}(s) = \frac{A_{open}(s)}{1+\beta(s)A_{open}(s)} = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})+\beta(s)A_0}$$

$$\equiv \frac{A_0}{(1+s/P\omega_{p1})(1+s/Q\omega_{p2})} = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})+sY}$$

1st order

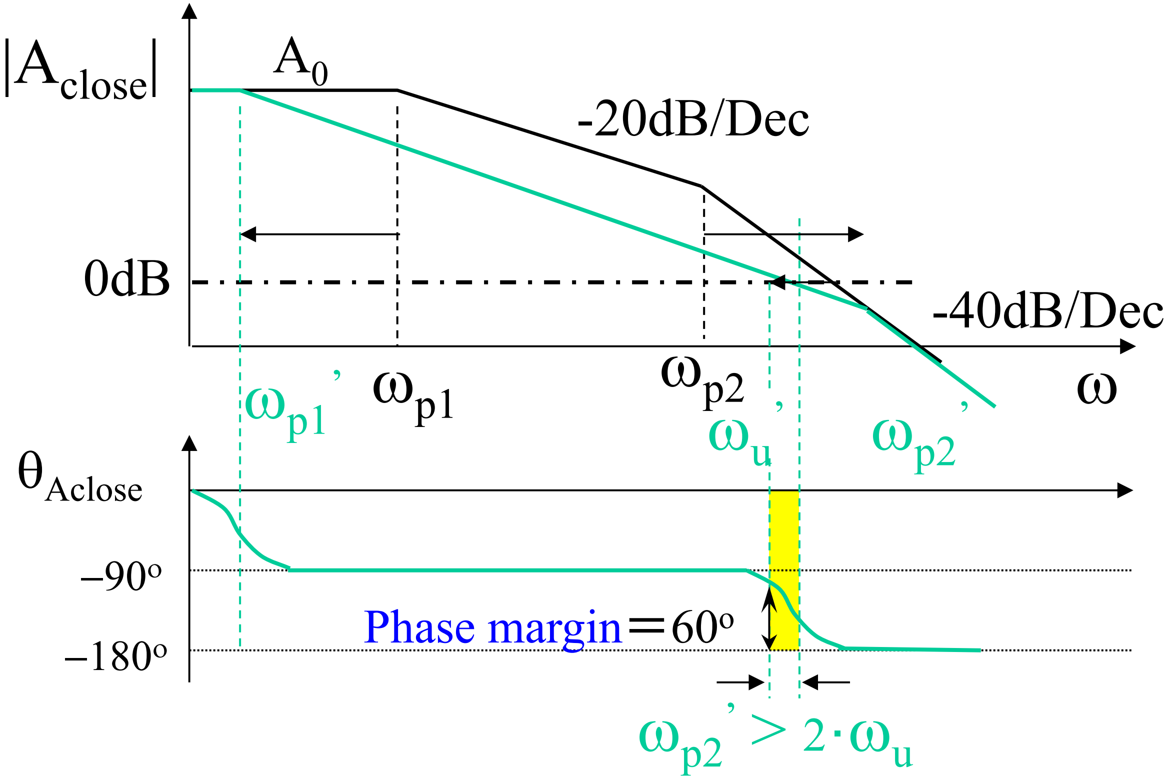
$$Y = (\frac{1}{P}-1)\frac{1}{\omega_{p1}} + (\frac{1}{Q}-1)\frac{1}{\omega_{p2}}$$

But the P and Q cannot set independently, because the constraint P･Q = 1 is imposed.

We can add the feedback loop to become β(s) = Y/A_0*s to separate two pole each other by ω_{p1} → P･ω_{p1}, ω_{p2} → Q･ω_{p2}.

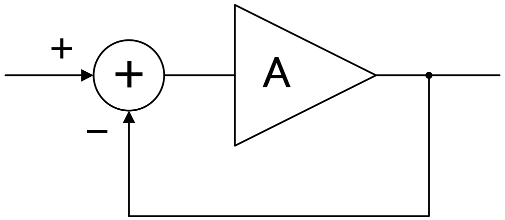
Phase compensation

Phase margin > 60° → Stability of the NFB loop is compensated for the unity gain configuration ($\beta = 1$).



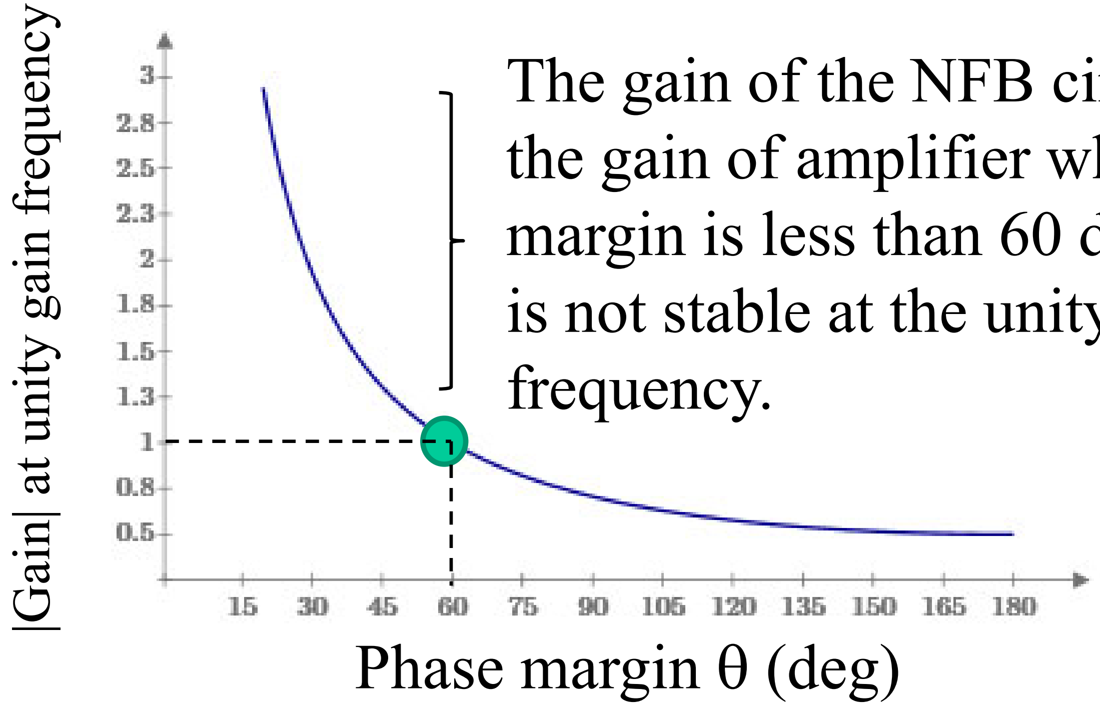
$\omega_{p2}' \geq 2 \cdot \omega_u'$ is required.

Phase margin of the voltage follower



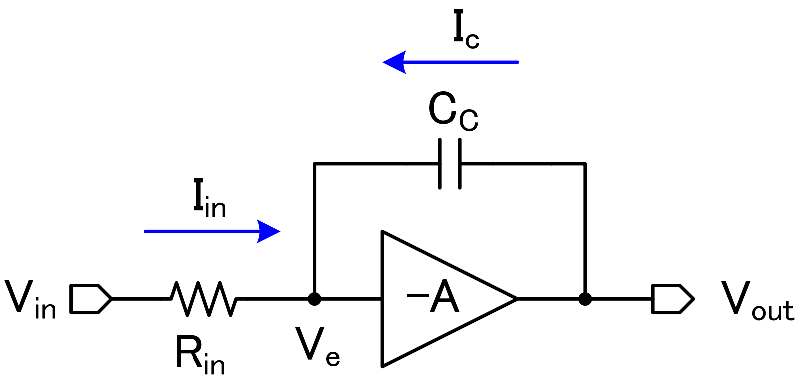
$$Gain = \frac{1}{1+\frac{1}{A}}$$

$$A = e^{j(\pi-\theta)}$$



The gain of the NFB circuit exceeds the gain of amplifier when the phase margin is less than 60 deg. This circuit is not stable at the unity gain frequency.

Design example (1)



$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

Phase compensation circuit of 2-pole amplifier

$$I_{in} = \frac{V_{in} - V_e}{R_{in}}$$

$$I_C = j\omega \cdot C_C(V_{out} - V_e) \cong j\omega \cdot C_C V_{out}$$

$$I_{in} + I_C = 0$$

$$\frac{V_{in} - V_e}{R_{in}} + j\omega \cdot C_C V_{out} = 0, \quad V_{out} = -AV_e$$

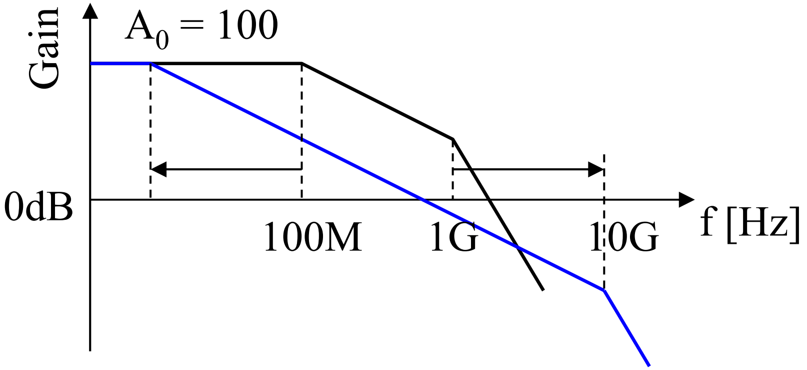
$$A_{Comp}(s) = \frac{V_{out}}{V_{in}} = \frac{-A}{1 + j\omega \cdot C_C R_{in} A}$$

$$= \frac{-A_0}{(1 + j\omega \cdot / \omega_{p1})(1 + j\omega \cdot / \omega_{p2}) + j\omega \cdot CcR_{in}A_0}$$

$$\equiv \frac{-A_0}{(1 + j\omega \cdot / \omega_{p1})(1 + j\omega \cdot / \omega_{p2}) + j\omega \cdot Y}$$

NOTE: By Miller effect, C_C is amplified A_0C_C, the time constant (Y) introduced by the compensation is enlarged by Y = $(A_0C_C)R_{in}$ with the small Cc.

Design example (2)



$$A_{Comp}(s) = \frac{-A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})+sC_CRA_0}$$

$$= \frac{-A_0}{(1+s/P\omega_{p1})(1+s/Q\omega_{p2})}$$

$$Y = (\frac{1}{P}-1)\frac{1}{\omega_{p1}}+(\frac{1}{Q}-1)\frac{1}{\omega_{p2}}$$

$$= C_CRA_0$$

If you set the parameters P = 1/10, Q = 10,

$$Y = (\frac{1}{P}-1)\frac{1}{\omega_{p1}}+(\frac{1}{Q}-1)\frac{1}{\omega_{p2}}$$

$$= \frac{10-1}{2\pi\cdot 100\text{M}}+\frac{0.1-1}{2\pi\cdot 1\text{G}} = 1.27\cdot 10^{-8}\text{(s)}$$

$$C_C = \frac{Y}{RA_0} = \frac{1.27\cdot 10^{-8}[\text{s}]}{100[\Omega]\cdot 100} = 1.27\text{(pF)}$$

$$\begin{cases} R_{in} = 100\ (\Omega) \\ C_C = 1.27\ (\text{pF}) \end{cases}$$